

NAAC - SSR IV CYCLE

# **M.Sc. MATHEMATICS**

**REGULATION 17** 

2019-20

**PROGRAMME STRUCTURE &** 

**SYLLABUS** 

Parvathaneni Brahmayya Siddhartha College of Arts & Science: Vijayawada-10.

(An Autonomous college in the jurisdiction of Krishna University)

Accredited at A+ grade by NAAC

# 2019 Batch M.Sc -Mathematics

	List of Courses				
C CODE	COURSE TITLE	CREDITS	TOTAL	CIA	SEE
	FIRST SEMESTER				
MA1T1	ALGEBRA	5	100	30	70
MA1T2	REAL ANALYSIS-I	5	100	30	70
MA1T3	ORDINARY DIFFERENTIAL EQUATIONS	5	100	30	70
MA1T4	TOPOLOGY	5	100	30	70
MA1T5	ADVANCED DISCRETE MATHEMATICS	5	100	30	70
	TOTAL	25	500	150	350
	SECOND SEMESTER				
MA2T1	GALOIS THEORY	5	100	30	70
MA2T2	REAL ANALYSIS-II	5	100	30	70
MA2T3	PARTIAL DIFFERENTIAL EQUATIONS	5	100	30	70
MA2T4	NUMERICAL METHODS WITH C	5	100	30	70
MA2T5	GRAPH THEORY	5	100	30	70
	TOTAL	25	500	150	350
	THIRD SEMESTER				
MA3T1	RINGS AND MODULES	5	100	30	70
MA3T2	COMPLEX ANALYSIS	5	100	30	70
MA3T3	FUNCTIONAL ANALYSIS-I	5	100	30	70
MA3T4	LATTICE THEORY	5	100	30	70
MA3T5	OPERATIONS RESEARCH-I	5	100	30	70
	TOTAL	25	500	150	350
AC01	ADVANCED MATHEMATICAL TECHNIQUES	2	50	50	-
	FOURTH SEMESTER				
MA4T1	NONCOMMUTATIVE RINGS	5	100	30	70
MA4T2	MEASURE & INTEGRATION	5	100	30	70
MA4T3	FUNCTIONAL ANALYSIS-II	5	100	30	70
MA4T4	ALGEBRAIC CODING THEORY	5	100	30	70
MA4T5	OPERATIONS RESEARCH-II	5	100	30	70
	TOTAL	25	500	150	350

# M.Sc. MATHEMATICS I SEMESTER MA1T1:ALGEBRA

Subject Code :	MA1T1	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives:** To develop skills and to acquire knowledge on some of the basic concepts in Group Theory, Ring theory and Vector Spaces.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	understand the properties of groups, rings and homomorphisms.
CO2	explain and apply Sylow's theorems.
CO3	understand the properties of ideals, quotient rings and integral domains.
CO4	illustrate the properties of Euclidean rings and polynomial rings.
CO5	know the concepts of vector spaces, dual spaces.

# Unit-I

**Group Theory**: Definition of a Group, Some Examples of Groups, Some Preliminary Lemmas, Subgroups, A Counting Principle, Normal Subgroups and Quotient Groups, Homomorphisms, Automorphisms, Cayley's theorem, Permutation groups. (2.1 to 2.10 of the prescribed book [1])

# Unit-II

**Group Theory Continued**: Another counting principle, Sylow's theorem, direct products, finite abelian groups (2.11 to 2.14 of the prescribed book [1])

# Unit-III

**Ring Theory**: Definition and Examples of Rings, Some special classes of Rings, Homomorphisms, Ideals and quotient Rings, More Ideals and quotient Rings, The field of quotients of an Integral domain (3.1 to 3.6 of the prescribed book [1])

# Unit-IV

**Ring Theory Continued**: Euclidean rings, A particular Euclidean ring, : Polynomial Rings, Polynomials over the rational field, Polynomial Rings over Commutative Rings (3.7 to 3.11 of the Prescribed book [1]).

# Unit-V

**Vector Spaces**: Elementary Basic Concepts - Linear Independence and Bases - Dual spaces (4.1 to 4.3 of the prescribed book [1]).

**PRESCRIBED BOOK:** [1] Topics in Algebra, I.N. HERSTEIN, Second Edition, Wiley Eastern Limited, New Delhi, 1988.

**REFERENCE BOOK:** "Basic Abstract Algebra", BHATTACHARYA P.B., JAIN S.K., NAGPAUL S.R., Second Edition, Cambridge Press,.

# M. Sc. DEGREE EXAMINATION MODEL QUESTION PAPER First Semester Mathematics Paper I – ALGEBRA

Time: Three hours

# Answer ONE question form each unit.Max Marks: 70All questions carry equal marks.

#### UNIT – I

- 1. (a) State and prove Lagrange's theorem. (CO1) (b) If H and K are finite subgroups of a group G of orders o(H) and o(K), respectively then prove that  $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$ . (CO1) (OR)
- 2. (a) Define Automorphism of a group G and prove that if G is a group then A(G), the set of automorphisms of a group G is also a group. (CO1)
  (b) Prove that every permutation is a product of two cycles. (CO1)

#### UNIT – II

3.	(a) State and Prove Cauchy's theorem.	(CO2)
	(b) State and Prove Sylow's third theorem.	(CO2)

#### (**OR**)

2. (a) Define Internal direct product of Groups. Let G be a group, suppose that G is the internal direct product of N<sub>1</sub>, N<sub>2</sub>, ...., N<sub>n</sub> and let T = N<sub>1</sub> x N<sub>2</sub> x .....x N<sub>n</sub> then prove that G and T are isomorphic. (CO2)
(b) State and Prove fundamental theorem on finitely generated abelian groups.(CO2)

#### UNIT – III

- 3. (a) Prove that a finite Integral domain is a field. (CO3)
  (b) Define an Ideal of a ring. If U is an Ideal of a ring R, then prove that R/U is a ring and it is a homomorphic image of R. (CO3)
  (OR)
- 4. (a) Define a Maximal Ideal of a ring R. If R is a commutative ring with unity and M is an ideal of R, then prove that M is Maximal if and only if R/M is a field.(CO3)
  (b) Prove that every Integral domain can be imbedded in a field. (CO3)

( **P.T.O.**)

#### UNIT – IV

5. (a) Define Euclidean ring. Prove that the ideal A=(a<sub>0</sub>) is a maximal ideal of the Euclidean ring R if and only if a<sub>0</sub> is a prime element of R. (CO4)
(b) Prove that J[i], the ring of Gaussian integers is a Euclidean ring. (CO4)

#### (**OR**)

6. (a) Define an irreducible polynomial over a field F and prove that the ideal A=(p(x)) in F[x] is a maximal ideal if and only if p(x) is irreducible polynomial over F.(CO4)
(b) State and Prove Gauss lemma. (CO4)

# $\mathbf{UNIT} - \mathbf{V}$

7. (a) Define vector space. If V is the internal direct sum of U<sub>1</sub>, U<sub>2</sub>..., U<sub>n</sub> then prove that V is isomorphic to the external direct sum of U<sub>1</sub>, U<sub>2</sub>,..., U<sub>n</sub>. (CO5)
(b) Prove that L(S), the linear span of S, is a subspace of the vector space V. (CO5)

#### (**OR**)

- 8. (a) If V is a finite dimensional vector space and if W is a sub space of V then prove that W is a finite dimensional, dim  $W \le \dim V$  and dim  $\frac{V}{W} = \dim V - \dim W$ . (CO5)
  - (b) If V and W are vector spaces of dimensions m and n, respectively, over F, then prove that Hom (V,W) is of dimension mn over F. (CO5)

# M.Sc. MATHEMATICS I SEMESTER MA1T2: REAL ANALYSIS-I

Subject Code :	MA1T2	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives:** To develop problem solving skills and to acquire knowledge on some of the basic concepts in limits, continuity, derivatives, the Riemann stieltjes – integrals and sequences of functions.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	determine the limits of functions and understand the properties of continuous functions.
CO2	describe the properties of differentiable functions and study the applications of the Mean Value theorems, L'Hospitals's rule and Taylor's theorem.
CO3	test the Riemann Stieltjes integrability of bounded functions and study its properties.
CO4	differentiate pointwise and uniform convergence of sequences of functions and illustrate the effect of uniform convergence on the limit function with respect to continuity, differentiability, and integrability.
CO5	understand the properties of equicontinuous functions and Stone Weierstrass theorem.

# UNIT-I

**Continuity**: Limits of functions, continuous functions, Continuity and Compactness, continuity and connectedness. Discontinuities, Monotonic functions, Infinite limits and limits at infinity. (4.1 to 4.34 of chapter 4)

# UNIT-II

**Differentiation**: Derivative of a real function, Mean value theorems, The continuity of derivatives, L'Hospital's rule, Derivatives of higher Order, Taylor's theorem, Differentiation of vector valued functions. (5.1to 5.19 of chapter5)

# UNIT-III

**Riemann Stieltjes Integral**: Definition and Existence of the Integral, Properties of the Integral, Integration and Differentiation, Integration of vector valued functions. Rectifiable curves. (6.1 to 6.27 of chapter 6)

# UNIT-IV

Sequences and series of functions: Discussion of main problem, Uniform convergence, Uniform convergence and Continuity, Uniform convergence and Integration, Uniform Convergence and Differentiation. (7.1 to 7.18 of chapter7)

# UNIT-V

**Sequences and series of functions(continued):** Equi continuous family of functions, Stone Weierstrass theorem.(7.19 to 7.33 of Chapter 7)

**PRESCRIBED BOOK:** "Principles of Mathematical Analysis", Third Edition, WALTER RUDIN, Tata Mc Graw Hill.

**REFERENCE BOOK:** "Mathematical Analysis", Second Edition, TOM .M. APOSTOL, Narosa Pub. 2002.

(CO2)

# M. Sc. DEGREE EXAMINATION MODEL QUESTION PAPER First Semester Mathematics

**Time: Three hours** 

# Paper II – REAL ANALYSIS -I Answer ONE question form each unit. All questions carry equal marks.

Maximum: 70 M

#### UNIT – I

- (a) Define limit of a function and continuous function. Show that a mapping f of a metric space X into a metric space Y is continuous on X if and only if f<sup>-1</sup>(V) is open in X, for every open set V in Y.
  - (b) Let X and Y be metric spaces and f is a mapping of E into Y,  $E \subset X$ , p is a limit point of E. Then show that  $\lim_{x \to n} f(x) = q$  if and only if  $\lim_{n \to \infty} f(p_n) = q$ , where

$$\{p_n\}$$
 is a sequence in E with  $p_n \neq p$  and  $\lim_{n \to \infty} p_n = p$  (CO1)

#### (**OR**)

- 2. (a) Define a continuous function. If f is a continuous mapping of a Compact Metric space X into a Metric space Y, then show that f is uniformly continuous on X.(CO1)
  - (b) If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X, then show that f(E) is connected. (CO1)

# UNIT – II

(a) If f and g are differentiable at some point x ∈ [a,b], then show that f+g, fg and f/g are differentiable at x and

(i) 
$$(f+g)'(x) = f'(x)+g'(x)$$
 (ii)  $(fg)'(x) = f'(x)g(x)+g'(x)f(x)$   
(iii)  $(f/g)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$  (CO2)

(b) If f is a real differentiable function on [a,b] and f'(a) < λ < f'(b) then show that there is x ∈ (a, b) such that f'(x) = λ.</li>
(OR)

- 4. (a) State and prove Taylor's theorem.
  - (b) If f is a continuous mapping of [a,b] into  $\mathbb{R}^k$  and f is differentiable in (a, b), then show that there is  $x \in (a, b)$  such that  $|f(b) - f(a)| \le (b-a) |f'(x)|$  (CO2)

#### UNIT – III

- 5. (a) Define Riemann integral of a function f(x) on [a, b]. Show that  $f \in R(\alpha)$  if and only if for every  $\varepsilon > 0$ , there exists a partition P such that U(P, f,  $\alpha$ ) L(P, f,  $\alpha$ ) <  $\varepsilon$ . (CO3)
  - (b) Suppose  $c_n \ge 0$  for  $n=1,2,..., \sum c_n$  converges,  $\{s_n\}$  is a sequence of distinct points in (a, b) and  $\alpha(x) = \sum_{n=1}^{\infty} c_n I(x - s_n)$ . If f is continuous on [a,b], then show that  $\int_a^b f d\alpha = \sum_{n=1}^{\infty} c_n f(s_n).$ (CO3) (OR)
- 6. (a) If  $f \in \mathbf{R}$  on [a,b] and if there is a differentiable function F on [a,b] such that F'=fthen show that  $\int_{a}^{b} f(x)dx = F(b) - F(a)$ . (CO3)
  - (b) Define Rectifiable curve. For any curve  $\gamma$ , if  $\gamma'$  is continuous on [a,b], then show that  $\gamma$  is rectifiable and  $\Lambda(\gamma) = \int_{a}^{b} |\gamma'(t)| dt$ . (CO3)

#### UNIT – IV

- 7. Let X be a Metric Space and E be a subset of X. (a) Suppose  $\lim_{n \to \infty} f_n(x) = f(x)$  on E. Put  $M_n = \sup_{x \in E} |f_n(x) - f(x)|$ . Then show that  $f_n \to f$  uniformly on E iff  $M_n \to 0$  as  $n \to \infty$ . (CO4)
  - (b) If  $\{f_n\}$  is a sequence of continuous functions on E and if  $f_n \rightarrow f$  uniformly on E then show that f is continuous on E. (CO4)
    - (**O**R)
- 8. (a) Let  $\alpha$  be a monotonically increasing function on [a, b]. Suppose  $f_n \in R(\alpha)$  for n=1,2,... and  $f_n \to f$  uniformly on [a, b]. Then show that  $f \in R(\alpha)$  on [a, b] and  $\int_{a}^{b} f d\alpha = \lim_{n \to \infty} \int_{a}^{b} f_n d\alpha$ (CO4)
  - (b) Show that there exists a real continuous function on the real line which is nowhere differentiable. (CO4)

 $\mathbf{UNIT} - \mathbf{V}$ 

- 9. (a) Define equicontinuous family of functions. If {f<sub>n</sub>} is a pointwise bounded sequence of complex functions on a countable set E, then show that {f<sub>n</sub>} has a subsequence {f<sub>nk</sub>} such that {f<sub>nk</sub>(x)} converges for every x ∈ E. (CO5)
  - (b) Let **B** be the uniform closure of an algebra **A** of bounded functions. Then show

that **B** is a uniformly closed algebra.

(CO5)

# (OR)

10. Let **A** be an algebra of real continuous functions on a compact set K. If **A** separates points on K and if **A** vanishes at no point of K, then show that the uniform closure **B** of **A** consists of all real continuous functions on K. (CO5)

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# M.Sc. MATHEMATICS I SEMESTER MA1T3: ORDINARY DIFFERENTIAL EQUATIONS

Subject Code :	MA1T3	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives :** To learn various methods for finding solutions of an ordinary differential equation(using Laplace transforms also) and to study the characteristics of solutions of differential equations.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	formulate and solve first order and second order differential equations.
CO2	determine power series and frobenius series solutions of first and second order differential equations.
CO3	solve linear systems of differential equations.
CO4	understand the properties of solutions of Legendre and Bessel Equations.
CO5	apply Laplace transforms to determine the solutions of differential equations.

# UNIT-I

**Second order linear equations**: Introduction, The general solution of the homogeneous equation, The use of a known solution to find another, The homogeneous equation with constant coefficients, The method of undetermined coefficients, The method of variation of parameters. (Sections 14 to 19 of Chapter 3 of [1])

# UNIT-II

**Power series solutions and special functions**: Introduction, A review of power series, Series solutions of first order equations, Second order Linear equations-Ordinary points, Regular singular points, Regular singular points (continued),

Gauss 's hyper geometric equation (Sections 26 to 31 of Chapter 5 of [1])

# UNIT-III

**Systems of linear differential equations**: Differential operators and an operator method, Applications, Basic theory of linear systems in normal form: Two equations in two unknown functions, Homogenous Linear Systems with constant coefficients: Two equations in two unknown functions (Sections 7.1. to 7.4 chapter 7 of [2])

# **UNIT-IV**

**Some special functions of Mathematical Physics**: Legendre polynomials, Bessels functions, The Gamma function, Properties of Bessel functions. (Sections 44 to 47 of chapter 8 of [1])

# UNIT-V

**Laplace Transforms**: Introduction, A few remarks on theory, Applications to differential equations, Derivatives and Integrals of Laplace transforms, Convolutions. (Sections 48 to 53 of chapter 9 of [1])

**PRESCRIBED BOOKS:** [1] " Differential equations with Applications and Historical Notes", Second edition , G.F. SIMMONS, Tata Mc Graw Hill , 2003.

[2] "Differential Equations", Third edition, SHEPLEY L ROSS, John Wiley & Sons, 1984

REFERENCE BOOK: "An Introduction to Ordinary Differential Equations", E.A. CODDINGTON, PHI

# M. Sc. DEGREE EXAMINATION MODEL QUESTION PAPER First Semester Mathematics

#### Paper III – ORDINARY DIFFERENTIAL EQUATIONS

Time: Three hours	Answer ONE question form each unit.	Maximum: 70
Μ		
	All questions carry equal marks.	Marks

#### **UNIT-I**

1.	(a) If $y_1$ and $y_2$ are are two linearly independent solutions of $y^{11}+P(x)y^1+Q(x)y^2$	=0 on
	[a,b], then show that $c_1y_1 + c_2y_2$ is the general solution on [a,b].	(CO1)
	(b) Solve the Euler's equidimensional equation $x^2y^{11}+3xy^1+10y=0$ .	(CO1)

#### (**OR**)

2. (a) Find the General Solution of y	$^{11}+4y = 2e^x + 4\cos 2x$	using	the	method	of
undetermined coefficients.					(CO1)
(b) Solve $xy^{11} - (1+x)y^1 + y = x^2 e^{2x}$	using the method of	variation	n of j	parameter	s.

(CO1)

#### **UNIT-II**

- (a)Define Ordinary Point and find the general Solution of (1+x<sup>2</sup>)y<sup>11</sup>+2xy<sup>1</sup>-2y =0 in terms of power series in x.
  - (b)Define Regular Singular Point and find the General solution of  $2x^2y^{11} + xy^1 (x+1)y = 0$ at the regular singular point x=0. (CO2)

#### (**OR**)

- 4. (a) Show that the equation  $x^2y^{11}$   $3xy^1$ +(4x+4)y =0 has only one Frobenius Series Solution. Find it. (CO2)
  - (b) Show that the equation  $4x^2y^{11}$   $8x^2y^1$ + $(4x^2+1)y=0$  has only one Frobenius Series Solution. Find the general solution. (CO2)

#### **UNIT-III**

5. Use the operator method to find the general solution of each of the following linear systems

$$\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t, \quad \frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$$
(CO3)

# (OR)

6. Find the general solution of the following linear system dx/dt = 5x+4y, dy/dt = -x+y (CO3)

#### **UNIT-IV**

7. (a) Derive Generating function for Legendre polynomials. (CO4)(b) Derive Rodrigue's formula for Legendre polynomials.

#### (**OR**)

8. (a) Show that the zeros of Bessel functions J<sub>p</sub>(x) and J<sub>p+1</sub>(x) occur alternately. (CO4)
 (b) Find the Bessel series of f(x) = x<sup>p</sup>, in terms of J<sub>p</sub>(λ<sub>n</sub>x), where λ<sub>n</sub> are the positive zeros of J<sub>p</sub>(x). (CO4)

#### **UNIT-V**

9. (a) Solve the differential equation xy<sup>11</sup> + (2x+3)y<sup>1</sup> + (x+3)y = 3 e<sup>-x</sup>, y(0) = 0 using Laplace transforms. (CO5)
 (b) Evaluate ∫<sub>0</sub><sup>∞</sup> e<sup>-ax</sup> sin bx/x / x dx, where a and b are positive constants. (CO5)

#### (**OR**)

10. (a) Show that  $L[x \cos ax] = \frac{p^2 - a^2}{(p^2 + a^2)^2}$  and use this result to find  $L^{-1}[\frac{1}{(p^2 + a^2)^2}]$ . (CO5) (b) Define Convolution and solve the Integral equation  $y(x) = e^x[1 + \int_0^x e^{-t} y(t) dt]$ using Laplace transforms. (CO5)

# M.Sc. MATHEMATICS I SEMESTER MA1T4: TOPOLOGY

Subject Code :	MA1T4	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives :**To generalize the concept of distance, open sets, closed sets and related theorems in real line and to learn basic concepts in Metric Spaces, Topological Spaces, compact spaces and connected spaces.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	understand the basic concepts of metric spaces and continuous functions.
CO2	discuss the properties of topological spaces, open bases and weak topologies.
CO3	understand the properties of compact spaces and describe Ascoli's theorem.
CO4	differentiate T1 and Haussdorff spaces, describe Uryzohn's lemma and Tietze extension theorem.
CO5	explain the concepts of connected spaces, components of a space and totally disconnected spaces.

# Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	L2				L2
CO2		L2			
CO3	L2				L2
CO4	L4				L2
CO5		L2			

# UNIT-I

**Metric Spaces**: The Definition and some examples, Open sets, Closed sets, Convergence, Completeness and Baire's theorem, Continuous mappings. (Sections 9 to 13 of chapter 2)

# UNIT-II

**Topological spaces** : The Definition and some examples, Elementary concepts, Open bases and Open subbases, Weak topologies.(Sections 16 to 19 of chapter 3)

# UNIT-III

**Compactness**: Compact spaces, Products of spaces, Tychonoff's theorem and locally Compact spaces, Compactness for Metric Spaces, Ascoli's theorem.(Sections 21 to 25 of chapter 4)

# UNIT-IV

Separation:  $T_1$  spaces and Hausdorff spaces, Completely regular spaces and normal spaces, Urysohn's Lemma and the Tietze extension theorem.(Sections 26 to 28 of Ch. 5)

# UNIT-V

**Connectedness**: connected spaces, The components of a space, Totally disconnected spaces. (sections 31 to 33 of chapter 6)

PRESCRIBED BOOK: "Introduction to Topology and Modern Analysis", G.F. SIMMONS, Mc. Graw Hill Book Company, New York International student edition.
 REFERENCE BOOKS: 1. "Topology", JAMES DUGUNDJI, Universal pub., 1990
 2. "General Topology", JOHN L KELLY, Springer, 2005

(MA1T4)

## M. Sc. DEGREE EXAMINATION MODEL QUESTION PAPER First Semester Mathematics

#### **Paper IV – TOPOLOGY**

Time: Three hours M	Answer ONE question form each unit.	Maximum: 70
	All questions carry equal marks.	Marks

#### UNIT – I

- 1. (a) Define a Metric space. Show that every non empty open set on the real line is the union of a countable disjoint class of open intervals. (CO1)
  - (b) Show that a subset F of a metric space is closed if and only if its complement F ' is open. (CO1)

#### (OR)

- (a) Define convergent sequence in a Metric Space. State and prove Cantor's intersection theorem. (CO1)
  - (b) Define a continuous function in a Metric Space. Show that a mapping f of a Metric space X into a Metric space Y is continuous ⇔ f<sup>-1</sup>(G) is open in X for every open set G in Y.

#### UNIT – II

- 3. (a) Define a Topological space . Let X be a topological space. If A is a subset of X, then show that A = { x /each neighborhood of x intersects A } (CO2)
  - (b) Let X be a topological space and A be a subset of X. Then show that (i)  $\overline{A} = A \cup D(A)$  and (ii) A is closed iff A contains D(A) (CO2)

# (OR)

4.	(a) State and Prove Lindelof's Theorem.	(CO2)
	(b) Show that every Separable Metric space is second countable.	(CO2)

#### UNIT – III

5.	(a) Define a Compact	space. Show that any closed subspace of a compact space	is
	compact.		(CO3)
	(b) State and prove Tyc	chonoff's theorem.	(CO3)

#### (**OR**)

6.	(a) Show that every Sequentially Compact Metric Space is Compact.	(CO3)
	(b) State and prove Ascoli's theorem.	(CO3)

# $\mathbf{UNIT} - \mathbf{IV}$

7 (a) Define a Hausdorff space. Show that the product of any non empty class of Hausdorff spaces is a Hausdorff space. (CO4)
 (b) Show that a one to one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism. (CO4)

(OR)

8.	(a) Show that every compact Haussdorff space is normal.	(CO4)
	(b) State and prove Urysohn's Lemma.	(CO4)

#### $\mathbf{UNIT} - \mathbf{V}$

9	(a)	Define a connected space. Show that a subspace of the real line is connected $\Leftrightarrow$	it is
		an interval.	(CO5)
	(b)	Show that any continuous image of connected space is connected.	(CO5)

# (OR)

- 10 (a) Let X be a topological space and A be a connected subspace of X. If B is a subspace of X such that  $A \subseteq B \subseteq \overline{A}$ , then show that B is connected. (CO5)
  - (b) Define a Totally disconnected space. Let X be a Hausdorff space. If X has an open base whose sets are also closed, then show that X is totally disconnected. (CO5)

# M.Sc. MATHEMATICS I SEMESTER MA1T5: ADVANCED DISCRETE MATHEMATICS

Subject Code :	MA1T5	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives :**To develop skills and to acquire knowledge on some of the basic concepts in Logic, Finite Machines, Lattices and their Applications.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	construct truth tables of statements and apply the rules of inference for conclusions.
CO2 understand the concept of finite machines.	
CO3	understand the basic concepts of algebraic structures, including lattices and Boolean algebras with examples.
CO4	determine the minimal forms of Boolean polynomials.
CO5	construct switching circuits and study the applications of switching circuits and Boolean algebras.

# Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1			L4		L3
CO2	L2				
CO3	L2				L2
CO4				L4	L3
CO5	L4				L4

# UNIT –I

**Logic :** Logic, Tautology, Normal Forms, Logical Inferences, Predicate Logic, Universal Quantifiers, Rules of Inference, Recurrence Relations, Solution using generating functions (1.6 to 1.10 of Chapter 1 & 3.7,3.8 of chapter 3 of [3])

# UNIT –II

**Finite Machines :** state machine, input-output machines, Introduction, state tables and state diagrams, simple properties, Dynamics, Behavior and Minimization. (Sections 5.1 to 5.5 of Chapter 5 of [1])

# UNIT – III

**Lattices:** Properties and Examples of Lattices, Distributive Lattices, Boolean Algebras. (Sections 1 to 3 of Chapter 1 of [2]).

# UNIT –IV

Lattices continued: Boolean polynomials, Ideals , filters and equations, Minimal forms of Boolean polynomials, (Sections 4,5,6 of Chapter -1 of [2])

# UNIT –V

**Application of Lattices**: Switching circuits, Applications of switching circuits, More Applications of Boolean Algebras (Sections 7, 8 and 9 of Chapter -2 of [2]).

**PRESCRIBED BOOKS** [1] "Application oriented Algebra" JAMES L FISHER, IEP, Dun-Downplay pub.1977.

[2] "Applied abstract algebra", Second Edition, R.LIDL AND G. PILZ, Springer, 1998.

[3] "Discrete Mathematical Structures", RM. SOMASUNDARAM, Prentice Hall of India, 2003

**REFERENCE BOOK:** "Discrete Mathematical Structures with Applications to Computer Science", J.P.TREMBLAY AND R.MANOHAR, Tata Mc. Graw Hill, 2002.

(MA1T5)

# M. Sc. DEGREE EXAMINATION MODEL QUESTION PAPER First Semester Mathematics Paper V – ADVANCED DISCRETE MATHEMATICS

Time: Three hoursAnswer ONE question form each unit.Maximum:70MarksAll questions carry equal marks.

#### UNIT – I

1	(a) Define a tautology. Show that the expression $((P^{\rightarrow}Q) \rightarrow R) \rightarrow (P \rightarrow (Q \lor R))$ is a
	tautology. (CO1)
	(b) Obtain DNF and CNF of the following formula
	$(\sim P \lor \sim Q) \rightarrow (P \leftrightarrow \sim Q). \tag{CO1}$
	(OR)

2 (a) Show that S v R is tautologically implied by  $(P v Q) \land (P \rightarrow Q) \land (Q \rightarrow S)$ . (CO1) (b) Solve  $a_r - 2a_{r-1} = (r+1)2^r$ . (CO1)

#### UNIT – II

- 3 (a)Define state machine congruence. Let f be a state homomorphism from the state machine M=(S,I,  $\delta$ ) onto the state machine M<sub>1</sub>=(S<sub>1</sub>, I,  $\delta$ <sub>1</sub>). Then show that there exists a state machine congruence on M such that M is isomorphic to M<sub>1</sub>. (CO2)
  - (b) Let  $M=(S, I, O, \delta, \theta)$  be an i/o machine and  $M_R$  be its reduced machine. If h is an i/o homomorphism from M onto  $M_1$ , then show that there exists an i/o homomorphism g from  $M_1$  onto  $M_R$  such that f = goh, where f is a natural homomorphism. (CO2)

# (OR)

4. Minimize the states of the following machine and write reduced machine. (CO2)

states		δ			θ
	0		1	0	1
1	2		5	1	0
2	2 5		5	1	1
3	1		8	1	1
4	8		2	1	0
5	6		5	1	1
1 2 3 4 5 6 7	1		5	1	1
7	2		5 5 8 2 5 5 5 3 5	1	0
8	3		5	1	1

# UNIT – III

5	<ul><li>(a) Define atom and join-irreducible element in a Lattice. Show that every atom irreducible.</li><li>(b) State and prove the distributive inequalities in Lattices.</li></ul>	is join- (CO3) (CO3)
	(OR)	
6	(a) State and prove De Morgan's laws in a Lattice.	(CO3)
	(b) State and prove Representation theorem in a Boolean Algebra.	(CO3)
	UNIT – IV	
7	<ul> <li>(a) Find DNF of the polynomial x(y+z)<sup>'</sup>+ (xy+z)<sup>'</sup>.</li> <li>(b) Define an ideal. Prove that an ideal M in a Boolean algebra B is maximal if an if for any b ∈ B either b ∈ M or b<sup>'</sup> ∈ M but not both.</li> </ul>	(CO4) d only (CO4)
	(OR)	
8	Minimize the following Boolean polynomial using Quiene- Mc Clusky method wx y z + w xy z' + wx y z' + w xyz + w x y z' + wxyz + w x yz + w x yz .	(CO4)
	$\mathbf{UNIT} - \mathbf{V}$	
9	(a) Draw the diagram for the following switching circuit $P = x_1(x_2(x_3+x_4)+x_3(x_5+x_6)).$	(CO5)

- (b) Determine the symbolic representation of the circuit given by  $P = (x_1+x_2+x_3)(x_1^{'}+x_2)(x_1x_2+x_1^{'}x_2)(x_2^{'}+x_3).$ (CO5) (CO5)
- 10 Explain the central lighting system in a room and draw its switching circuit. (CO5)

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# M.Sc. MATHEMATICS II SEMESTER MA2T1: GALOIS THEORY

Subject Code :	MA2T1	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives :** To develop skills and to acquire knowledge on some of the basic concepts in Modules, Algebraic Extensions, Splitting fields, Polynomials solvable by radicals.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Understand the concepts of modules, submodules, quotient modules, homomorphisms and characterize
COI	completely reducible modules.
CO2	Derive and apply Gauss Lemma, Eisenstein criterion for irreducibility of Polynomials.
CO3	Demonstrate Field extensions and characterization of finite normal extensions as splitting fields and
003	study prime fields.
CO4	Derive Fundamental theorem of Galois theory, fundamental theorem of Algebra and related results.
CO5	Understand cyclotomic polynomials, cyclic extensions, Radical field extensions and Ruler & Compass
005	Constructions.

# Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	L2				L2
CO2			L3		L3
CO3	L2				
CO4					L3
CO5	L4				

# UNIT-I

**Modules:** Definition and examples, Sub modules and direct sums, R-homomorphisms and quotient modules, Completely reducible modules. (Sections 1 to 4 of chapter 14 of [1]) **UNIT-II** 

**Algebraic extensions of fields:** Irreducible polynomials and Eisenstein's criterion, Adjunction of roots, Algebraic extensions, Algebraically closed fields.(Sections 1 to 4 of Chapter15 of [1]) **UNIT-III** 

**Normal and separable extensions:** Splitting fields, Normal extensions, multiple roots, Finite fields, Separable extensions.(Sections 1 to 5 of Chapter16 of [1]) UNIT-IV

**Galois Theory :** Automorphism groups and fixed fields, Fundamental theorem of Galois Theory, Fundamental theorem of Algebra. (Sections 1 to 3 of Chapter 17 of [1]) **UNIT-V** 

**Applications of Galois theory to classical problems:** Roots of unity and cyclotomic polynomials; Cyclic extensions; Polynomials solvable by radicals; Ruler and compass constructions.(Sections 1,2,3,5 of Chapter 18 of [1])

PRESCRIBED BOOK: [1]. "Basic Abstract Algebra", Second Edition, BHATTACHARYA P.B., JAIN S.K., NAGPAUL S.R., Cambridge Press.

#### **REFERENCE BOOKS:**

"Galois Theory", Second Edition, JOSEPH ROTMAN, Springer, 1998
 "Algebra", ARTIN M, PHI, 1991

(MA2T1)

#### M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Second Semester Mathematics Paper I – GALOIS THEORY

Time: Three hours

Answer ONE question form each unit. Maximum: 70 Marks All questions carry equal marks.

#### **UNIT-I**

1. (a) Define an R-module and a submodule. If  $(N_i)$ ,  $1 \le i \le k$ , is a family of R-submodules

of a module M then prove that,  $\sum_{i=1}^{k} N_i = \{x_1 + x_2 + ... + x_k / x_i \in N_i\}$ . (CO1, BTL-3)

(b) Define an R- homomorphism of a module. Let f be an R- homomorphism of an R-module M in to an R- module N, then prove that M/ ker  $f \cong f(M)$ . (CO1, BTL-3)

(**OR**)

- 2. (a) Let R be a ring with unity then prove that an R-module M is cyclic if and only if  $M \cong R/I$ , for some left ideal I of R. (CO1, BTL-4)
  - (b) Define simple module. Let R be a ring with unity and let M be an R- module. Then prove that the following statements are equivalent. (CO1, BTL-4)
    - (i) M is simple
    - (ii)  $M \neq (0)$  and M is generated by any  $0 \neq x \in M$
    - (iii)  $M \cong R/I$ , where I is a maximal left ideal of R.

#### **UNIT-II**

3. (a) Define root of a polynomial. State and prove Gauss lemma. (CO2, BTL-3)

(b) Define an extension field. Let p(x) be an irreducible polynomial in F[x]. Then prove that there exists an extension E of F in which p(x) has a root. (CO2, BTL-3) (OR)

4. (a) Define an algebraic element and algebraic extension of afield. If E is a finite extension of a field F then prove that E is an algebraic extension of F.

(CO2, BTL-2)

(b) Define algebraically closed field and algebraic closure of a field. Let F be a field. Then prove that there exists an algebraically closed field K containing F as a subfield.

(CO3, BTL-4)

#### **UNIT-III**

- 5. (a) Define splitting field of a polynomial. State and prove the uniqueness of a splitting field. (CO3,BTL-2)
  - (b) Prove that the degree of the extension of the splitting field of  $x^3-2 \in Q[x]$  is 6.

#### (**OR**)

6. (a) Define normal extension of a field and a prime field. Prove that any finite field F with

p<sup>n</sup> elements is the splitting field of  $x^{p^n} - x \in F_p[x] \in (CO3,BTL-4)$ 

- (b) Define separable extension of a field. Let E be an extension of a field F then prove that the following are equivalent. (CO3,BTL-2)
  - (i)  $E=F(\alpha)$  for some  $\alpha \in E$
  - (ii) There are only a finite number of intermediate fields between F and E.

#### **UNIT-IV**

7. (a) Define fixed field of a group of automorphisms. State and prove Dedikind lemma.

(CO4, BTL-2)

(b) If  $f(x) \in F[x]$  has r distinct roots in its splitting field E over F, then prove that the Galois group G(E/F) of f(x) is a subgroup of the symmetric group  $S_r$ . (CO4, BTL-2)

#### (**OR**)

Define Galois extension of a field. State and prove the fundamental theorem of Galois theory. (CO4, BTL-2)

#### **UNIT-V**

- 9. (a) Define primitive n<sup>th</sup> root of unity. Let F be a field and let U be a finite subgroup of the Multiplicative group  $F^* = F \{0\}$ . Then prove that U is cyclic. (CO5, BTL-3)
  - (b) Define cyclic extension of a field. Let F contain a primitive n<sup>th</sup> root of unity. Then prove that the following are equivalent.
    - (i) E is a finite cyclic extension of degree n over F.
    - (ii) E is the splitting field of an irreducible polynomial  $x^n b \in F[x]$ .(CO5, BTL-3)

#### (**OR**)

10. (a) Define radical extension of a field. Prove that the polynomial  $f(x) \in F[x]$  is solvable by radicals if and only if its splitting field E over F has solvable Galois group G(E/F).

(CO5, BTL-3)

- (b) If a and b are constructible numbers, then prove that
  - (i) ab is constructible.
  - (ii) a/b,  $b\neq 0$ , is constructible. (CO5, BTL-5)

# M.Sc. MATHEMATICS II SEMESTER MA2T2: REAL ANALYSIS-II

Subject Code :	MA2T2	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives:** To develop problem solving skills and to acquire knowledge on some of the basic concepts in Power Series, Functions of Several Variables, the Inverse function theorem, Implicit function theorem, Differential forms and their integration.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Understand the properties of power series, Fourier series, Exponential, Trigonometric and Logarithmic
COI	functions.
CO2	Compute derivatives and integrals of real valued and vector valued functions of several variables.
CO3	Understand and apply the inverse function theorem, implicit function theorem.
CO4	Understand the concept of Differential forms, their product, the exterior derivative
CO5	Apply Stokes's theorem and study the concepts related to simplexes and their integrals

# Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	L2				
CO2					L3
CO3	L4				L3
CO4	L2				
CO5	L3				

# UNIT-I

# Some special functions:

Power series, The exponential and logarithmic functions, The trigonometric functions, The algebraic completeness of the complex field, Fourier series. (Sections 8.1 to 8.15 of Chapter8) **UNIT-II** 

**Functions of several variables:** Linear Transformations, Differentiation, Contraction Principle. (Sections 9.1 to 9.23 of Chapter 9)

# **UNIT-III**

# Functions of several variables (Continued):

Inverse function theorem, Implicit function theorem, The Rank theorem, determinants, derivatives of higher order and differentiation of integrals. (Sections 9.24 to 9.43 of Chapter 9) **UNIT-IV** 

**Integration of differential forms:** Integration, Primitive mappings, partitions of unity, change of variables, differential forms.(10.1 to 10.25 of Chapter10)

# UNIT-V

# Integration of differential forms (continued):

Simplexes and chains, Stoke's theorem, closed forms and exact forms.(10.26 to 10.41 of chapter 10)

PRESCRIBED BOOK: "Principles of Mathematical Analysis", Third Edition,

#### WALTER RUDIN, Tata Mc Graw Hill.

#### **REFERENCE BOOKS**:

1. "Mathematical Analysis", TOM .M. APOSTOL, Second Edition, Narosa Pub. 2002.

2. "A First Course in Mathematical Analysis", D. SOMASUNDARAM, B. CHOUDARY, Narosa Publishing House.

(MA2T2)

# M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Second Semester Mathematics Paper II – REAL ANALYSIS-II

**Time: Three hours** 

Answer ONE question form each unit. All questions carry equal marks. Maximum: 70 Marks

#### UNIT -I

5. (a) Suppose 
$$\sum_{n=0}^{\infty} c_n$$
 converges. Let  $f(\mathbf{x}) = \sum_{n=0}^{\infty} c_n \mathbf{x}^n$  (-1< x < 1). Then show that  

$$\lim_{x \to 1} f(\mathbf{x}) = \sum_{n=0}^{\infty} c_n .$$
(CO1, BTL-4)

(b) Suppose  $f(x) = \sum_{n=0}^{\infty} c_n x^n$ , the series converges in |x| < R. If -R < a < R, then show

that f can be expanded in a power series about the point x = a which converge in

$$|x-a| < R - |a|$$
 and  $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$ , if  $|x-a| < R - |a|$ . (CO1, BTL-4)

#### (OR)

- 2 Show that
  - (a) The function E is periodic, with period  $2\Pi i$ .
  - (b) The functions C and S are periodic, with period  $2\Pi$ .
  - (c) If  $0 \le t \le 2\Pi$ , then  $E(it) \ne 1$
  - (d) If z is complex number with |z| = 1, there is a unique t in  $[0,2\pi)$  such that E(it)=z. (CO1, BTL-3)

# UNIT -II

- 3 (a) Show that a linear operator A on a finite dimensional vector space X is one -to one if and only if the range of A is all of X. (CO2, BTL-3)
  - (b) Suppose f maps an open set  $E \subset R^n$  into  $R^m$ . Then show that  $f \in \mathfrak{E}'(E)$  if and only

if the partial derivatives  $D_{j}f_{i}$  exists and are continuous on E for  $1 \le i \le m, 1 \le j \le n$ (CO2, BTL-4)

(OR)

4. (a) Suppose f maps a convex open set  $E \subset R^n$  into  $R^m$ , f is differentiable in E, and there is a real number M such that  $||f'(x)|| \le M$  for every  $x \in E$ . Then show that

$$|f(b) - f(a)| \le M|b - a| \text{ for all } a \in A, b \in E.$$
(CO2, BTL-3)

- (b) Define a contraction mapping. State and prove contraction principle. (CO2, BTL-3) UNIT- III
- 5 State and prove inverse function theorem. (CO3, BTL-4) (OR)

6 (a) If A and B are n by n matrices then show that det  $([B][A]) = \det [B] \det [A]$ . (CO3, BTL-3)

(b) Suppose f is defined in an open set  $E \subset R^n$ , suppose that  $D_1 f$ ,  $D_{21} f$  and  $D_2 f$  exists at every point of E, and  $D_{21} f$  is continuous at some point  $(a,b) \in E$ . Then show that  $D_{12} f$  exists and  $(D_{12} f)(a,b) = D_{21} f(a,b)$ 

#### (CO3, BTL-3)

#### **UNIT -IV**

# 7 (a) For every f ∈ X (I<sup>k</sup>), Show that L(f)= L<sup>1</sup>(f) (CO4, BTL-4) (b) Suppose F is a X ' mapping of an open set E ⊆ R<sup>n</sup> into R<sup>n</sup>, 0 ∈ E, F(0)=0 and F '(0) is invertible. Then show that there exists a neighborhood of0 in R<sup>n</sup> in which a representation F(x)= B<sub>1</sub>...B<sub>n-1</sub>G<sub>n</sub> o....oG<sub>1</sub>(x) is valid where each G<sub>i</sub> is a primitive X '- mapping in some neighborhood of 0 ; G<sub>i</sub>(0)=0, G<sub>i</sub>'(0) is invertible and each B<sub>i</sub> is either a flip or the identity operator. (CO4, BTL-3)

(OR)

8 (a) Suppose K is a compact subset of R<sup>*n*</sup> and {V<sub> $\alpha$ </sub> /  $\alpha \in \Delta$ } is an open cover of K.

Prove that there exists functions  $\psi_{1,}\psi_{2,}...\psi_{s} \in X(\mathbb{R}^{n})$  such that

- (i)  $0 \le \psi_i \le 1$ , where  $1 \le i \le s$
- (ii) Each  $\psi_i$  has it's support in some  $V_{\alpha}$

(iii) 
$$\psi_1(x) + \psi_2(x) + \dots + \psi_s(x) = 1$$
, for every  $x \in K$ . (CO4, BTL-3)

(b) Suppose E is an open set in R<sup>n</sup>. If w is of class X" in E, then show that d<sup>2</sup> w = 0. (CO4, BTL-4)

#### UNIT -V

9. (a) Define an affine chain, simplex. If  $\sigma$  is an oriented rectilinear k-simplex in an open set

 $E \subset \mathbb{R}^{n} \text{ and if } \overline{\sigma} = \varepsilon \text{ } \sigma \text{ then show that } \int_{\overline{\sigma}}^{} w = \varepsilon \int_{\sigma}^{} w \text{ for every k-form } w \text{ in E.(CO5, BTL-4)}$ (b) State and prove Stoke's theorem.
(CO5, BTL-4)
(OR)
10. (a) Define an exact form. If  $E \subset \mathbb{R}^{n}$  is convex open set, if  $k \ge 1$ , if w is k- form of class X '

in E, and if dw = 0, then show that there is a (k-1)- form  $\lambda$  in E such that  $w = d\lambda$ . (CO5, BTL-4)

(b) Fix 1≤ k ≤n, let E ⊂ R<sup>n</sup> be an open set in which every closed k- form is exact and let T be a one – one X "- mapping of E onto an open set U⊂ R<sup>n</sup> whose inverse is also of class X ". Then show that every closed k-form in U is exact in U. (CO5, BTL-4)

# M.Sc. MATHEMATICS II SEMESTER MA2T3: PARTIAL DIFFERENTIAL EQUATIONS

Subject Code :	MA2T3	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives :** To develop problem solving skills and to acquire knowledge on some of the basic concepts in partial differential equations and to learn essential methods for finding solutions of classical partial differential equations.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Formulate and classify first order and second order partial differential equations
CO2	Solve the first order linear and non linear equations using different methods
CO3	Solve the wave equation with different initial and boundary conditions and can apply these solutions to
005	physical problems
CO4	Solve the Laplaces equation with different initial and boundary conditions and can apply these
004	solutions to physical problems
CO5	Solve the Heat equation with different initial and boundary conditions and can apply these solutions to
005	physical problems

# Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1					L5
CO2	L4				L4
CO3				L3	
CO4				L3	
CO5				L3	

# UNIT-I

First Order Partial Differential equations: Curves and Surfaces, Genesis of first order partial differential equations, Classification of Integrals, Linear equations of the first order, Pfaffian Differential equations, Compatible systems. (Sections 1.1 to 1.6 of Chapter 1 of [1]).

# UNIT-II

Charpit's method, Jacobi's Method, Integral surfaces through a given curve. (Sections 1.7to1.9 of Chapter 1 of [1]).

# UNIT-III

Second order Partial differential Equations: Genesis of Second Order Partial Differential Equations. Classification of Second Order Partial differential equations One Dimensional Waves equation, Vibrations of an infinite string, Vibrations of a semi infinite string. Vibrations of a string of Finite Length, Riemann's Method, Vibrations of a string of finite length (Method of separation of variables.)(Sections 2.1 to 2.2 of [1] & Sections 2.3.1 to 2.3.5 of Chapter 2 of [1] ).

## **UNIT-IV**

Laplace's Equation: Boundary value problems, Maximum and Minimum principles, The Cauchy problem, The Dirichlet problem for the upper Half plane, The Neumann problem for the upper Half plane, The Dirichlet Interior problem for a circle, The Dirichlet Exterior problem for a circle, The Neumann Problem for a circle, The Dirichlet problem for a Rectangle. (Sections 2.4.1 to 2.4.9 of Chapter 2 of [1]).

#### **UNIT-V**

Harnacks Theorem, Laplaces Equation-Green's Function. The Dirichlet problem for a Half plane, The Dirichlet problem for a circle, Heat conduction-Infinite Rod case, Heat conduction-Finite Rod case, Duhamel's principle, Wave equation, Heat Conduction Equation. (Sections 2.4.10to2.4.13, 2.5.1to2.5.2, 2.6.1to2.6.2 of Chapter 2 of [1]).

#### **PRESCRIBED BOOK:**

[1] An Elementary course in Partial Differential Equations, T.AMARANATH, Second Edition, Narosa Publishing House, 2003

#### **REFERENCE BOOKS:**

[2] "Elements of Partial Differential Equations", SNEEDON IAN, Tata Mc Graw Hill, 1987

[3]. "Introduction to Partial Differential Equations", K. SANKARA RAO, PHI, 2003

(MA2T3)

# **M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Second Semester Mathematics Paper III – PARTIAL DIFFERENTIAL EQUATIONS**

Time: Three hours

Answer ONE question form each unit. All questions carry equal marks.

Maximum: 70 Marks

#### UNIT-I

1. (a)Define General Integral and Find the General Solution of  $2x(y+z^2)p + y(2y+z^2)q =$  $z^3$ 

(CO1, BTL-4)

(b)Show that the Necessary and Sufficient condition for the Pfaffian differential

equation X. dr = P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0 to integrable is X. Curl X =0

(CO1, BTL-4)

2. (a)Find the integral of  $yz dx + (x^2y - zx) dy + (x^2z - xy) dz = 0$ . (CO1, BTL-4)

(b)Define Complete Integral and Find a complete integral of  $p^2x + q^2y = z$  (CO1, BTL-4)

(OR)

#### UNIT-II

- 3. (a) Solve  $xu_x + yu_y = (u_z)^2$  by Jacobi's Method. (CO2, BTL-4)
  - (b) Find the Integral surface of the equation  $(2xy -1)p + (z 2x^2)q = 2(x-yz)$  which passes through the line  $x_0(s) = 1$ ,  $y_0(s) = 0$  and  $z_0(s) = s$ . (CO2, BTL-4)

#### (OR)

- 4. (a) Find the Integral surface of the equation p<sup>2</sup>x + pqy =2pz + x, passing through the line y=1, x=z
   (CO2, BTL-4)
  - (b) Reduce the equation  $u_{xx} + x^2 u_{yy} = 0$  to a canonical form. (CO2, BTL-4)

#### UNIT-III

- 5. (a) Find the d'Alembert's Solution for the problem of Vibrations of a string of infinite length. (CO3, BTL-4)
  - (b) Prove that for the equation  $u_{xy} + \frac{1}{4}u = 0$ , the Riemann function is  $v(x,y, \alpha, \beta) = J_0(\sqrt{(x-\alpha)(y-\beta)})$  (CO3, BTL-4)

#### (OR)

- 6. (a) Solve the Problem of vibrations of a string of finite length using the method of separation of variables. (CO3, BTL-4)
  - (b)Show that the solution of the problem of vibrations of string finite length is unique. (CO3, BTL-

4)

#### **UNIT-IV**

7. Solve the Dirichlet interior problem for a circle. (CO4, BTL-4) (OR)

8.	<ul><li>(a) Solve the Dirichlet problem for a Rectangle.</li><li>(b) Solve the Neumann problem for the upper Half Plane.</li></ul>	(CO4, BTL-4) (CO4, BTL-4)
	UNIT-V	
9.	<ul><li>(a)State and Prove Harnack's Theorem.</li><li>(b) Solve the Dirichlet problem for a Half plane using Green's function.</li><li>(OR)</li></ul>	(CO5, BTL-4) (CO5, BTL-4)
10.	<ul> <li>(a) Solve Heat Conduction problem in a rod of infinite length.</li> <li>(b) Solve u<sub>tt</sub> - c<sup>2</sup> u<sub>xx</sub> = F(x,t), 0&lt; x &lt; 1, t &gt; 0 U(x,0)= f(x), U<sub>t</sub>(x, 0)= g(x)</li> </ul>	(CO5, BTL-4) (CO5, BTL-4)

 $\begin{array}{l} U_t(x,0)=g(x),\\ U(0,t)=u_t(l,t)=\!0,\\ \mbox{by making use of Duhamel's Principle.} \end{array}$ 

# M.Sc. MATHEMATICS II SEMESTER MA2T4: NUMERICAL METHODS WITH C

Subject Code :	MA2T4	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives:** To study the concept of interpolation and to learn numerical methods for differentiation, integration. To develop numerical solutions of differential equations and linear systems and also to solve problems using programming language C.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Explore alternate algorithmic approaches to problem solving
CO2	Choose data types and structures, unions to solve mathematical and scientific problems
CO3	Compare the viability of different approaches to the numerical solution of problems arising in interpolation and approximation and analyze the error incumbent in any such numerical approximation.
CO4	Evaluate a derivative at a value using an appropriate numerical method and calculate the value of a definite integral using an appropriate numerical method.
CO5	Derive and apply numerical methods like single step methods, multi step methods to solve the linear system of equations and analyze the error incumbent in any such numerical approximation.

# Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
<b>CO1</b>		L2			
CO2			L3		
CO3	L4				
CO4					L5
CO5					L3

# UNIT – I

C - Basics, C - Character set, Data types ,Variables, Constants, Expressions, Structure of C program, Operators and their precedence and Associativity, Basic input and output statements, Control structures, Simple programs in c using all the operators and control structures.

# Functions

Concept of a function, Parameters and how they are passed, Automatic Variables, Recursion, Scope and extent of variables, Writing programs using recursive and non – recursive functions (1.4,1.7,1.11,1.12 of Chapter 1, 2.2,2.3,2.4 of Chapter 2, 3.1,3.2,3.3 of Chapter 3 & 5.1, 5.2,5.3 of Chapter 5 of [1])

UNIT – II Arrays and Strings Single and multidimensional Arrays, Character array as a string, Functions on strings, Writing C Programs using arrays and for string manipulation.

# Pointers

Pointers declarations ,Pointers expressions, Pointers as parameters to functions, Pointers and Arrays, Pointer arithmetic.

# Structures & Unions

Declaring and using structures, Operations on structures, Arrays of structures, User defined data types, Pointers to Structures.

(4.1 to 4.6 of Chapter 4, 6.1 to 6.8 of Chapter 6, Chapter 9 & Chapter 10 of [1])

# **UNIT-III**

**Interpolation and Approximation:** Introduction, Lagrange and Newton Interpolations, Finite difference Operators, Interpolating polynomials using finite differences, Hermite Interpolations. (Section 4.1 to 4.5 of chapter 4 of [2]).

# UNIT-IV

**Numerical Differentiation and Integration:** Introduction, Numerical differentiation ,Numerical integration, Methods based on Interpolation, Methods based on Undetermined Coefficients, Composite Integration Methods.

(Sections 5.1, 5.2, 5.6, 5.7, 5.8, 5.9 of chapter 5 of [2])

# UNIT-V

**Ordinary Differential Equations:** Introduction, Numerical Method, Single step Methods, Multistep methods.(sections 6.1 to 6.4 of chapter 6 of [2])

PRESCRIBED BOOKS: [1] " C Programming: a practical approach", AJAY MITTAL,

Pearson Publications.

[2] "Numerical Methods for Scientific and Engineering Computation", M.K.JAIN,S.R.K. IYANGAR AND R.K. JAIN Third edition, New Age International (p) Limited, New Delhi, 1997.

**REFERENCE BOOKS: 1.** "Numerical Analysis", P.C. BISWAL, PHI, 2008 2. "Computer Oriented Numerical Methods", V. RAJA RAMAN, Third Edition, PHI.

(MA2T4)

**Max.: 70** 

# M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Second Semester Mathematics Paper IV – NUMERICAL METHODS WITH C

Time:	Three	hours
Marks		

# Answer ONE question form each unit.

All questions carry equal marks.

#### UNIT -I

1	<ul><li>(a) Write notes on data types supported by C.</li><li>(b) Write a program to find standard deviation of a given sequence. (OR)</li></ul>	(CO1, BTL-2) (CO1, BTL-3)
2	<ul><li>(a) write a note on operator precedence and associativity.</li><li>(b) Write a program to check given string is polindrome or not.</li></ul>	(CO1, BTL-3) (CO1, BTL-3)

# UNIT -II

3	(a) Explain single dimensional arrays.	(CO2, BTL-2)
	(b) Write a program to find n fibonocci numbers using functions.	(CO2, BTL-3)
	(OR)	
4	(a) Distinguish between structures and unions. Define pointers.	(CO2, BTL-4)

4	(a) Distinguish between stru	ctures and unions. Define pointers	$\mathbf{S}. \qquad (\mathbf{CO2}, \mathbf{B1L-4})$
	(b) Write a program to find	the sum of complex numbers.	(CO2, BTL-3)

# **UNIT-III**

5	(a) Derive Newton's divided difference interpolation formula.	(CO3, BTL-4)
	(b) Find the unique polynomial of degree 2 or less such that $f(0) = 1, \pm 1$	f(1)=3, f(3)=55
	using the Newton-Divided Difference interpolation.	(CO3, BTL-4)

# (OR)

-						
6	(a)	Derive	Lagrange's	interpolation	formula	(CO3, BTL-4)
0	(4)		Dugrunge b	merpolation	Tormana.	(000, D11)

(b) For the following data calculate the difference table and obtain the forward difference polynomial and interpolate at x = 0.25 and x = 0.35. (CO3, BTL-4)

х	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.00	2.28

# UNIT –IV

- 7 (a) Derive Gauss- Legendre three point formula for numerical integration. (CO4, BTL-4)
  - (b) Evaluate the integral  $I = \int_{-1}^{1} (1-x^2)^{3/2} \cos x \, dx$  using Gauss- Chebyshev three point formula. (CO4, BTL-5)

(OR)

8 Find an approximate value of  $I = \int_{0}^{1} \frac{dx}{1+x}$  using

(a) Trapezoidal rule (b) Simpson's rule . Obtain a bound on the error. (CO4, BTL-4)

#### UNIT -V

- 9 (a) Derive a formula for Euler's method to solve an initial value problem and it's error. (CO5, BTL-4)
  (b) Solve the I.V.P. u'= 2tu<sup>2</sup>, u(0)=1, with h=0.2 on the interval [0,1] using the
  - back ward Euler method. (CO5, BTL-4)

#### (OR)

10 (a) Derive second order Runge-Kutta method for solving initial value problem.

(CO5, BTL-3)

(b) Solve the system

u' = -3u + 2vv' = 3u - 4v, u(0) = 0, v(0) = 0.5

with h= 0.2 on the interval [0,0.4] using the Euler- Cauchy method. (CO5, BTL-4)

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# M.Sc. MATHEMATICS II SEMESTER MA2T5: GRAPH THEORY

Subject Code :	MA2T5	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives:** To develop skills and to acquire knowledge on some basic concepts in connected graphs, Euler graphs, Hamiltonian graphs, Trees and Circuits, Planar graphs and Dual graphs etc.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Understand the basic concepts of graphs, directed graphs, and weighted graphs, Euler and Hamiltonian graphs and able to present a graph by matrices.
CO2	Understand the properties of trees and able to find a minimal spanning tree for a given weighted graph.
CO3	Illustrate the properties of cut-sets and cut-vertices in graphs.
CO4	Demonstrate the properties of planar graphs and dual graphs and detect the planarity of a given graph
CO5	Illustrate the structure of a graph as vector space and understand the applications of various types of graphs in real life.

# Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	L2				L2
CO2	L4				L4
CO3					L3
CO4				L3	
CO5				L3	

# UNIT-I

**Introduction:** What is a Graph, Finite and Infinite graphs, Incidence and degree, Isolated Vertex, Pendant Vertex and Null Graph.

**Paths and circuits:** Isomorphism, Subgraphs, a puzzle with multi colored cubes. walks, Paths and Circuits, connected graphs, Disconnected graphs, Components, Euler graphs, Operations on graphs, More on Euler graphs, Hamiltonian paths and circuits, Travelling – Salesman Problem. (Chapters 1 and 2 of [1]).

# UNIT-II

**Trees and Fundamental Circuits:** Trees, some properties of trees, pendant Vertices in a tree, distances and centers in a tree, rooted and binary trees, on Counting trees, spanning trees, fundamental circuits, finding all spanning trees of a graph, spanning trees in a weighted Graphs. (Chapter 3 of [1])

# UNIT-III

**Cut sets and Cut –vertices:** Cut sets, Some Properties of a Cut Set, All cut sets in a Graph, Fundamental circuits and cut sets, connectivity and separability, network flows, 1-isomorphism, 2- isomorphism. (Chapter 4 of [1])

# **UNIT-IV**

**Planar and dual graphs:** Combinatorial Vs Geometric graphs, Planer graphs, Kuratowski's two graphs, Different representations of a planar graph, Detection of planarity, Geometric dual. (Sections 1 to 6 of Chapter 5 of [1])

#### **UNIT-V**

**Vector spaces of a graph:** Sets with one operation, Sets with two operations, Modular arithmetic and Galois field, Vectors and Vector spaces, Vector space associated with a graph, Basis vectors of graph, circuits and cut-set sub spaces. (Sections 1 to 7 of Chapter 6 of [1])

#### **PRESCRIBED BOOK:**

[1] "Graph theory with applications to Engineering and Computer Science", NARSINGH DEO, Prentice Hall of India Pvt., New Delhi,1993.

#### **REFERENCE BOOK:**

"Graph Theory with Applications", BONDY J.A AND U.S.R. MURTHY, North Holland,

(MA2T5)

M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Second Semester Mathematics Paper V – GRAPH THEORY

Time: Three hours

Answer ONE question form each unit. Maxie All questions carry equal marks.

Maximum: 70 Marks

#### UNIT-I

1. (a) If graph G has exactly two vertices of odd degree, then show that the	re must be
a path joining these two vertices.	(CO1, BTL-4)
(b) Show that a simple graph with n vertices and k components can have ½(n-k)(n-k+1) edges.	at most (CO1, BTL-4)
(OR)	
2. (a) Prove that a connected graph G is Euler graph if and only if all vertices	of G are of
even degree.	(CO1, BTL-4)
(b) Define a Hamilton circuit. Prove that a complete graph with n vertices have	½(n-1)
edge-disjoint Hamilton circuits, if n is an odd number ≥3.	(CO1, BTL-4)
UNIT-II	

1.	(a) Define a tree. Prove that there is one and only one path between every	pair of
	vertices in a tree.	(CO2, BTL-4)
	(b) Prove that every tree contains at least two pendent vertices.	(CO2, BTL-4)

#### (**OR**)

(a) Define a spanning tree. Show that every connected graph has at least one spanning tree.
 (CO2, BTL-4)

(b) Define a shortest spanning tree. Show that a spanning tree is a shortest spanning tree if and only if there exists no other spanning tree at a distance of one from T whose weight is smaller than that of T. (CO2, BTL-4)

#### UNIT-III

(a) Define a cut set. Show that every cut set in a connected graph G must contains at least one branch of every spanning tree of G. (CO3, BTL-4)
 (b)Show that every circuit has an even number of edges in common with any cut set. (CO3, BTL-4)

#### (OR)

6. (a) Define a cut vertex. Show that a vertex v in a connected graph G is a cut vertex if and only if there exists two vertices x and y in G such that every path between x and y posses through v. (CO3, BTL-4)

(b) Define the edge connectivity of a graph. Show that the edge connectivity of a graph can never exceed the degree of the vertex with smallest degree in G. (CO3, BTL-4)

#### **UNIT-IV**

7. (a) Define a planar graph. Show that the complete graph with five vertices is non planar. (CO4, BTL-4)
(b) Show that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane. (CO4, BTL-4)

#### (**OR**)

	(OK)	
8.	(a) State and prove Euler's formula.	(CO4, BTL-4)
	(b) How to detect the given graph is planar or non- planar.	(CO4, BTL-4)

#### **UNIT-V**

9.	(a) Prove that the ring sum of two circuits in a graph G is either a circuit	or an edge
	disjoint unions of circuits	(CO5, BTL-4)
	(b) Prove that the set consisting of all the cut sets and the edge disjoint u	nions of cut
	sets in a graph G is an abelian group under the ring sum operation.	(CO5, BTL-4)

## (OR)

10. (a) Prove that in a graph G, W<sub>G</sub> is a vector space. (CO5, BTL-4)
(b) Prove that the set of all circuit vectors in W<sub>G</sub> forms a sub space of W<sub>S</sub>. (CO5, BTL-4)

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# M.Sc. MATHEMATICS III SEMESTER MA3T1: RINGS AND MODULES

Subject Code :	MA3T1	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives:** To develop skills and to acquire knowledge on some advanced concepts of Modern Algebra i.e, different algebraic structures like Modules, Prime ideals in commutative rings, complete ring of quotients, Prime ideal spaces, functional representations of elements of a ring.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:	
CO1	Understand the concepts of Ring theory, Boolean algebras and Boolean rings.	
CO2	Study classical isomorphism theorems and properties of direct sum, product of rings.	
CO3	Understand the concept of Prime ideals and Radicals in commutative rings	
CO4	Study the Wedderburn –Artin theorem and its Applications	
CO5	Study functional representations and Prime ideal spaces.	

Mapping of Course Outcomes:

	<b>PO1</b>	PO2	PO3	PO4	PO5
CO1	L3				
CO2			L4		
CO3	L4				
CO4					L3
CO5			L5		

# UNIT-I (18 Hours) (Online Mode)

# Fundamental Concepts of Algebra:

Rings and related Algebraic systems, Subrings, Homomorphisms, Ideals. (Sections 1.1, 1.2 of chapter 1)

# UNIT-II (18 Hours)

# Fundamental Concepts of Algebra:

Modules, Direct products and Direct sums, Classical Isomorphism Theorems. (Sections 1.3, 1.4 of chapter 1)

# UNIT-III (18 Hours)

# Selected Topics on Commutative Rings:

Prime ideals in Commutative Rings, Prime ideals in Special Commutative Rings. (Sections 2.1, 2.2 of Chapter 2)

## **UNIT-IV (18 Hours)**

Selected Topics on Commutative Rings: The Complete Ring of Quotients of a Commutative Ring, Ring of quotients of Commutative Semi Prime Rings. (Sections 2.3, 2.4 of Chapter 2)

UNIT-V (18 Hours)

Selected Topics on Commutative Rings: Prime Ideal Spaces (Section 2.5 of chapter 2) Appendices: Functional Representations (Appendix 1: Proposition 1 to Proposition 9)

PRESCRIBED BOOK: "Lectures on Rings and Modules", J. Lambek, Blaisdell Publications.

**REFERENCE BOOK: "Algebra",** Thomas W. Hungerford, Springer publications.

MA3T1)

# M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Third Semester Mathematics Paper I – RINGS AND MODULES

Time:3 hours	Answer ONE question form each unit.	Maximum : 70
marks	All questions carry equal marks	,

#### UNIT I

(a) Define Boolean Algebra and Boolean ring. Show that a Boolean algebra becomes a complemented distributive lattice by defining avb=(a 'Λ b ')' & 1=0' and conversely, any complemented distributive lattice is a Boolean algebra in which these equations are provable identities. (CO1)

(b) In a Boolean algebra, show that a''=(a')'=a.

(CO1)

#### (**OR**)

- 2 (a) Define congruence relation and homomorphic relation. Show that there is a one-one correspondence between the ideals K and the congruence relations  $\Theta$  of a ring R such that r-r'  $\varepsilon K \Leftrightarrow$  r  $\circ$  r' and this is an isomorphism between the lattice of ideals and the lattice of congruence relations. (CO1)
  - (b) If A,B and C are additive subgroups of a ring R, then show that (i) (AB)C = A(BC)(ii)  $AB \subset C \Leftrightarrow$  $A \subset C : B \Leftrightarrow B \subset A \cap C$ (CO1)

#### **UNIT II**

3 Show that the following statements are equivalent. (a) R is isomorphic to a finite direct product of rings  $R_i$  (i =1,2, ...,n)

(b)There exist central orthogonal idempotents  $e_i \in \mathbb{R}$  such that  $1 = \sum_{i=1}^{n} e_i$ ,  $e_i \mathbb{R} \cong \mathbb{R}_i$ (CO2)

(c) R is a finite direct sum of ideals  $K_i \cong R_i$ 

#### (OR)

(a) Define Artinian and Noetherian Modules. let B be a sub module of  $A_{R}$ . Then show that A 4 is Artinian if and only if B and A/B are Artinian. (CO2) (b) If e is a central idempotent in a ring R, then show that eR is indecomposable if and only

if e is an atom of B(R). (CO2)(P.T.O.)

(MA3T1)

#### **UNIT III**

5 (a) Define prime radical and Jacobson radical. show that the radical of a ring R consists of all elements  $r \in R$  such that 1 - r x is a unit for all  $x \in R$ . (CO3)

(b)Show that every ring is a sub direct product of sub directly irreducible rings. (CO3)

#### (OR)

- 6 Define a prime ideal and regular ring. Let R be a sub directly irreducible commutative ring with smallest non zero ideal J. Then show that
  - (a) The annihilator  $J^*$  of J is the set of all zero divisors
  - (b)  $J^*$  is a maximal ideal and  $J^{**} = J$ .

#### (CO3)

#### **UNIT IV**

7 If R is any commutative ring, then show that Q(R) is rationally complete. (CO4)

- 8 (a) If R is a commutative ring, then show that Q(R) is regular iff R is semi prime. (CO4)
  - (b) If R is commutative semi prime and rationally complete, then show that every annihilator of R is a direct summand. (CO4)

#### UNIT V

- 9 (a) Show that a Boolean algebra is isomorphic to the algebra of all subsets of a set if and only if it is complete and atomic. (CO5)
  - (b) If R is any atomic Boolean algebra , show that its completion is isomorphic to the algebra of all sets of atoms of R. (CO5)

#### (OR)

- 10 (a) If P is a prime ideal of the commutative ring R, then show that  $P/0_{P}$  is a prime ideal of  $R/0_{P}$  and it contains all zero divisors of  $R/0_{P}$ . (CO5)
  - (b) If P is any subset of the commutative ring R whose complement is closed under finite products, then show that  $R/0_{P} \subset R^{P}/O^{P} \subset (R/0_{P})^{P/0_{P}}$ , upto isomorphism. (CO5)

# M.Sc. MATHEMATICS III SEMESTER MA3T2: COMPLEX ANALYSIS

Subject Code :	MA3T2	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives :** To acquire knowledge on basic concepts of complex analysis and to learn methods for finding integrals of complex valued functions.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Represent complex numbers and understand the concept of limits and continuity for complex valued
COI	functions.
CO2	Use the Cauchy-Riemann equations to find the derivative of a complex valued function and able to find
002	power series representations of analytic functions.
CO3	Apply Cauchy integral formula to evaluate complex contour integrals
CO4 Classify singularities and evaluate complex integrals using the residue theorem.	
CO5	Understand Maximum Modulus Principle and study their applications

## Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	L3				
CO2			L4		
CO3					L5
CO4			L3		
CO5					L5

## **UNIT-I (18 Hours) (Online Mode)**

**The Complex Number system**: The real numbers, The Field of Complex Numbers, The Complex plane, Polar representation and roots of Complex numbers, Lines and Half planes in the Complex plane, The extended plane and it's spherical representation.

**Elementary Properties and Examples of Analytic Functions**: Power series, Analytic functions. Analytic functions as mappings, Mobius transformations. (Chapters I and III).

## UNIT-II (18 Hours)

**Complex Integration**: Power series representation of Analytic functions, Zeros of an Analytic function, The Index of a closed curve (Sections 2, 3 and 4 of chapter IV)

## UNIT-III (18 Hours)

**Complex Integration** :, Cauchy's Theorem and Integral formula, The homotopic version of Cauchy's theorem and simple connectivity, Counting zeros , The open mapping theorem. The Goursat's Theorem (Sections 5 to 8 of Chapter IV)

## **UNIT-IV(18 Hours)**

Singularities: Classification of Singularities, Residues, The Argument Principle.(ChapterV)

## UNIT-V(18 Hours)

**The Maximum Modulus Theorem**: The Maximum Principle, Schwarz's Lemma, Convex functions and Hadamard's three circle Theorem, Phragmen-Lindelof theorem.(Chapter VI).

**PRESCRIBED BOOK:** "Functions of one Complex Variable", Second Edition, John B. Conway, Springer International Student Edition, Narosa publishing House.

#### **REFERENCE BOOKS:**

**1. "Complex Variables and Applications",** sixth Edition, James ward Brown and Ruel V. Churchill, Mc Graw Hill International Editions.

2 "Foundations of Complex Analysis", . S. Ponnusamy, Narosa Publishing House.

(MA3T2)

# M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Third Semester Mathematics Paper II – COMPLEX ANALYSIS

Time: 3 hrs

Answer ONE question form each unit. Maximum: 70 marks All questions carry equal marks.

# UNIT I

1. (a) State and Prove Chain Rule.

(CO1)

(b)Show that any power series and its derivative have same radius of convergence. (CO1)

## (OR)

- 2 a) Let u and v be real valued functions defined on a region G and suppose that u and v have continuous partial derivatives. Then show that f: G→C defined by f(z)= u(z)+i v(z) is analytic if and only if u and v satisfy the Cauchy –Riemann Equations. (CO1)
  - b) Show that a Mobius Transformation takes circles onto circles. (CO1)

#### **UNIT II**

3 State and Prove Leibniz's Rule.

#### (OR)

4 (a)State and prove Fundamental Theorem of Algebra. (CO2)

(b)Show that the Zeros of an Analytic function defined on a open connected set are isolated. (CO2)

#### **UNIT III**

5	(a) State and Prove Cauchy's Integral formula (First Version).	(CO3)

(b) State and Prove Morera's Theorem.

#### (OR)

(MA3T2)

(CO2)

(CO3)

- 6 (a) If G is Simply connected and f: G → C is analytic in G, then show that f has a primitive (CO3)
  (b) Suppose f is analytic in B(a,R) and Let α =f(a). If f(z)- α has a Zero of order m at z=a,
  - then show that there is C>0 and  $\delta >0$  such that for I  $\zeta -\alpha I < \delta$ , the equation  $f(z) = \zeta$  has exactly m simple roots in  $B(a,\varepsilon)$ . (CO3)

#### **UNIT IV**

7 (a) State and Prove Casorati weierstrass theorem. (CO4)

(b) Show that 
$$\int_0^{\pi} \frac{d\theta}{a + \cos \theta} = \frac{\pi}{\sqrt{a^2 - 1}}$$
 (CO4)

#### (OR)

8 (a) Find $\int_{-\infty}^{\infty} \frac{x^2 dx}{1+x^4}$	(CO4)
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(b) State and prove Residue theorem . (CO4)

# UNIT V

9 (a) State and Prove Maximum Modulus theorem.	(CO5)
(b) State and Prove Schwarz's Lemma.	(CO5)

# (OR)

10 (a) Show that a function f on [a, b] is convex iff $f'$ is increasing.	(CO5)
(b) State and Prove Phragmen Lindelof Theorem.	(CO4)

# M.Sc. MATHEMATICS III SEMESTER MA3T3: FUNCTINAL ANALYSIS- I

Subject Code :	MA3T3	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives :** To develop skills and to acquire knowledge on basic concepts of Banach Spaces, Hilbert Spaces, operators.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:		
CO1	Review the fundamental properties of Metric spaces, Normed and Banach Spaces		
CO2	Explain the behaviour of linear operators, bounded and continuous operators on finite dimensional normed spaces.		
CO3	Understand concepts of Hilbert spaces and construct orthonormal sequences and series using Gran Schmidt process.		
CO4	Understand Riesz representation theorem, self adjoint, unitary operators on Hilbert spaces and applications to bounded linear functional.		
CO5	Study the applications of Hahn-Banach theorem, open mapping theorem, uniform boundedness theorem		

Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1			L3		
<b>CO2</b>		L3			
CO3				L3	
<b>CO4</b>	L3				
CO5					L5

## UNIT-I (18 Hours) (Online Mode)

Review of Properties of Metric spaces, Vector spaces, Normed spaces, Banach spaces, Further properties of Normed spaces, Finite Dimensional normed spaces and Sub spaces, Compactness and Finite Dimension.

(Chapter-1 and 2.1 to 2.5 of Chapter 2)

# UNIT-II(18 Hours)

Linear Operators, Bounded and Continuous Linear Operators, Linear Functionals, Linear Operators and Functionals on Finite Dimensional Spaces, Normed Spaces of Operators, Dual space.

 $(2.6 \mbox{ to } 2.10 \mbox{ of Chapter } 2$  )

# UNIT -III(18 Hours)

Inner product spaces, Hilbert Space, Further properties of Inner product spaces, Orthogonal

Complements and Direct sums, Orthonormal sets and sequences, Series related to Ortho normal sequences and sets. (Sections: 3.1 to 3.5 of Chapter 3)

# UNIT – IV(18 Hours)

Total orthonormal sets and sequences, Representation of functionals on Hilbert Spaces, Hilbert- Adjont Operator, Self-Adjoint, Unitary and Normal operators. (Sections: 3.6 to 3.10 of Chapter 3)

## UNIT-V (18 Hours)

Zorn's Lemma, Hahn Banach Theorem, Hahn Banach Theorem for Complex Vector Spaces and Normed Spaces, Applications to Bounded Linear Functonals of C[a,b], Adjoint Operators, Reflexive spaces. (4.1 to 4.6 of Chapter 4)

**PRESCRIBED BOOK:** "Introductory Functional analysis with Applications", Erwin Kreyszig, John Wiley & Sons.

**REFERENCE BOOK: "Functional Analysis- A First Course", M. Thamban Nair, PHI** 

(MA3T3)

# M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Third Semester Mathematics Paper III – FUNCTIONAL ANALYSIS -I

Time: 3 hours Marks	Answer ONE question form each unit.	Maximum: 70
	All questions carry equal marks.	
	UNIT I	

#### UNIT I

1. a) State and prove Minkowski's inequality.

(CO1)

b) Define a Banach space.	Show that R <sup>n</sup> is a Banach space.	(CO1)

(OR)

2. a) Show that every finite dimensional subspace of a normed space is complete. (CO1)

b) Show that a compact subspace M of a metric space is closed and bounded. (CO1)

#### **UNIT II**

3. a) Prove that if a normed space X is finite dimensional, then every linear operator on X is bounded. (CO2)
b) Prove that a linear operator T : D(T)→Y is continuous if and only if T is bounded.

#### (OR)

4. a) Prove that a finite dimensional vector space is algebraically reflexive. (CO2)b)Prove that if Y is a Banach space, then B(X,Y) is a Banach space. (CO2)

#### **UNIT III**

5. a) State and prove Schwarz inequality on an Inner product space. (CO3)

b) If X is an inner product space and M is a non empty convex subset of X which is complete,

then prove that for any  $x \in X$ , there exists a unique  $y \in M$  such that  $\delta = \inf_{\widetilde{y} \in M} ||x - \widetilde{y}|| = ||x - y||.$ (CO3)

#### (**OR**)

6 a) State and prove Bessel inequality. (CO3) b) Show that for any subset  $M \neq \phi$  of a Hilbert space H, M is dense in H if and only if  $M^{\perp} = \{0\}.$  (CO3)

(MA3T3)

#### **UNIT IV**

7	a) Prove that every orthonormal set in any separable Hilbert space is countable.	(CO4)
	b) State and prove Riesz's theorem.	(CO4)

## (OR)

8 a) Define Hilbert adjoint operator. Show that the Hilbert adjoint operator T<sup>\*</sup> of a bounded

linear operator T is also a bounded linear operator with the norm  $||T^*|| = ||T||$  (CO4)

b) Define a self adjoint operator. Show that a bounded linear operator T:  $H \rightarrow H$ , where H

is

(CO2)

## an Hilbert space, then $\langle Tx, x \rangle$ is real for all x in H. (CO4)

#### UNIT V

- 9 a) State and prove Hahn Banach Theorem for normed spaces. (CO5)
  - b) If X is a normed space and  $x_0 \neq 0$  be any point on X then show that there exists a bounded

linear functional  $\bar{f}$  on X such that  $\|\bar{f}\| = 1 \& \bar{f}(x_0) = \|x_0\|$  (CO5)

# (OR)

10 a) Define an adjoint operator. Show that the adjoint operator T\* of a bounded linear operator T

is also linear and bounded with  $||T^*|| = ||T||$  (CO5)

b) Prove that every finite dimensional normed space is reflexive.

(CO5)

# M.Sc. MATHEMATICS III SEMESTER MA3T4A: LATTICE THEORY

Subject Code :	MA3T4A	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives :** To develop skills and knowledge of standard concepts in Hass diagrams, complete lattices, distributive lattices, Boolean algebras and classical propositional logic.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:		
CO1	Understand partially ordered sets and Jordan Dedekind chain conditions		
CO2 Analyze the relationship between posets and lattices, acquire knowledge of fundamental noti			
002	lattice theory		
CO2	Define and understand basic properties of complete lattices and conditionally complete lattices,		
closure operations and their applications.			
CO4	Characterize modular and distributive lattices using the Birkhoff and Dedekind criterions		
CO5	Understand Boolean algebras, Boolean rings and lattices of relations and propositions		

Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	L3				
CO2			L4		
<b>CO3</b>	L3				
CO4				L4	
CO5			L3		

## UNIT –I (18 Hours) (Online Mode) Partly Ordered Sets:

Set Theoretical Notations, Relations, Partly Ordered Sets, Diagrams, Special Subsets of a Partly Ordered Set, Length, Lower and Upper Bounds, The Minimum and Maximum Conditions, The Jordan–Dedekind Chain Condition, Dimension Functions.

(Sections 1 to 9 of chapter I)

# UNIT – II (18 Hours)

# Lattices in General:

Algebras, Lattices, The Lattice Theoretical Duality Principle, Semi Lattices, Lattices as Partly Ordered Sets, Diagrams of Lattices, Sub Lattices, Ideals, Bound Elements of a Lattice, Atoms And Dual Atoms, Complements, Relative Complements, Semi Complements, Irreducible and Prime Elements of a Lattice, The Homomorphism of a Lattice, Axiom Systems of Lattices. (Sections 10 to 21 of chapter II)

# UNIT – III (18 Hours)

# **Complete Lattices:**

Complete Lattices, Complete Sub Lattices of a Complete Lattice, Conditionally Complete

Lattices, Compact Elements and Compactly Generated Lattices, SubAlgebra Lattice of an Algebra, Closure Operations, Galois Connections, Dedekind Cuts, Partly Ordered Sets as Topological Spaces. (Sections 22 to 29 of chapter III)

## UNIT – IV (18 Hours)

## **Distributive and Modular Lattices:**

Distributive Lattices, Infinitely Distributive and Completely Distributive Lattices, Modular Lattices, Characterization of Modular and Distributive Lattices by their Sublattices, Distributive Sub lattices of Modular Lattices, The Isomorphism Theorem of Modular Lattices, Covering Conditions, Meet Representation in Modular and Distributive Lattices. (Sections 30 to 36 of chapter IV)

## UNIT-V (18 Hours)

# **Boolean Algebras:**

Boolean Algebras, De Morgan Formulae, Complete Boolean Algebras, Boolean Algebras and Boolean Rings, The Algebra of Relations, The Lattice of Propositions, Valuations of Boolean Algebras. (Sections 42 to 47 of chapter VI)

PRESCRIBED BOOK: "Introduction to Lattice Theory", Gabor Szasz, Academic press.

**REFERENCE BOOK: "Lattice Theory", G. Birkhoff, Amer. Math.Soc.** 

(MA3T4A)

## M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Third Semester Mathematics Paper IV – LATTICE THEORY Answer ONE question form each unit. All questions carry equal marks.

**Time: Three hours** 

#### UNIT I

- 1. (a) Define a Completely ordered set . If every chain of a partly ordered set P has an upper bound, then prove that P contains a maximal element. (CO1)
  - (b) Prove that a partly ordered set can satisfy both the maximum and minimum conditions if and only if every one of its sub chain is finite. (CO1)

- 2. (a)Show that every sub chain of a partly ordered set satisfying the maximum condition has a greatest element. (CO1)
  - (b)Define JDCC. Let P be a partly ordered set bounded below, locally finite length and satisfies JDCC . Prove that there exists a dimension function on P. (CO1)

## **UNIT II**

- 3. (a) Define an order isomorphism. Show that if two lattices are isomorphic if and only if they are also order isomorphic. (CO2)
  - (b) In a lattice show that the relation defined by  $a \le b$  if and only if a  $\Lambda b=a$  is an ordering relation. (CO2)

#### (OR)

- 4. (a) (i) Show that every weakly complemented lattice is semi complemented.
  - (ii) Show that every section complemented lattice bounded below is weakly complemented. (CO2)
  - (b) Define meet irreducible element. Show that in a lattice satisfying the maximum condition, every one of its elements can be represented as the meet of a finite number of meet irreducible elements. (CO2)

## UNIT III

- 5. (a) Define a Complete Lattice. If a lattice satisfies both the maximum and minimum conditions then show that it is complete. (CO3)
  - (b) Prove that every order preserving mapping of a complete lattice into itself has a fix element. (CO3)

#### (OR)

- 6. (a) Show that every element of a compactly generated lattice can be represented as a meet of finite number of meet irreducible elements. (CO3)
  - (b)Define a closure operation. Let  $\phi$  be a closure operation of a partly ordered set P. If R consists of  $\phi$ -closed elements of P and if  $\inf_P R$  exists, then show that this infimum is also  $\phi$ -closed. (CO3)

#### **UNIT IV**

7. Define Modular lattice, Distributive lattice. State and Prove Dedekind's modularity criterion. (CO4)

#### (OR)

8. (a)Show that all irredundant irreducible meet - representations of any element of a modular lattice have the same number of components. (CO4)
(b)Show that every modular lattice satisfies the double covering condition. (CO4)

## UNIT V

- 9. For a complete Boolean algebra B, show that the following conditions are equivalent.
  - (a) B is completely meet- distributive.
  - (b) B is atomic.
  - (c) B is isomorphic with the subset lattice of a set.

# (OR)

10. (a) Show that the algebra of relations R (M) of a set M forms a complete Boolean algebra.

(CO5)

(CO5)

(b) Let S be any Boolean sub algebra of a Boolean  $\sigma$  - algebra A. Then Show that the set M of all measurable elements of A is a Boolean sub algebra of A including S. (CO5)

# M.Sc. MATHEMATICS III SEMESTER MA3T4B: SEMIGROUPS

Subject Code :	MA3T4B	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives :** To acquire knowledge on basic concepts of Semi groups, Semi lattices, congruences, Regular Semi groups, Simple Semi groups and Completely 0- Semi Simple groups.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Know the basic definitions in semi group theory
CO2	Construct new semigroups using congruences
CO3	Understand the basic properties of Green's relations, regular semi groups
CO4	Understand the definitions of 0-simple semigroups and the proofs of some of the main theorems in this section
CO5	Understand the concepts of congruences on completely 0-simple semigroups, free semi groups

# Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	L3				
CO2				L4	
CO3	L3				
CO4			L3		
CO5			L4		

# UNIT – I (18 Hours) (Online Mode)

Basic Definitions, Monogenic Semigroups, Ordered Sets, Semilattices and Lattices, Binary Relations, Equivalences. (Sections 1 to 4 of Ch. I)

# UNIT – II (18 Hours)

Congruences, Free Semigroups, Ideals and Rees Congruences, Lattices of Equivalences And Congruences. ( Sections 5 to 8 of Ch. I )

# UNIT - III (18 Hours)

Introduction, The equvivalences  $\mathcal{L}, \mathcal{R}, \mathcal{H}, \mathcal{J}$  and  $\mathcal{D}$ , The structure of  $\mathcal{D}$ -Classes, Regular Semigroups. (Chapter II)

# UNIT -IV (18 Hours)

Introduction, Simple and 0 – Simple Semigroups, Principle Factors, Rees's Theorem, Primitive Idempotents. (Sections 1 to 3 of Chapter III )

# UNIT -V (18 Hours)

Congruences on Completely 0 – Simple semigroups, The Lattice of Congruences on a Completely 0 – Simple Semigroup, Finite Congruence- Free Semigroups. (Sections 4 to 6 of Chapter III)

**PRESCRIBED BOOK: "An Introduction to Semigroup Theory",** J.M. Howie, Academic Press.

**REFERENCE BOOK: "The Algebraic Theory of Semigroups",** A.H. Clifford, G.B. Preston, American Mathematical Society.

(MA3T4B)

# M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Third Semester Mathematics Paper IV – SEMI GROUPS

Time: 3 hours

Answer ONE question form each unit. Maximum: 70Marks All questions carry equal marks.

# UNIT –I

- 1 (a) Show that a semi group S with zero is a o- group if and only if  $(\forall a \in S/\{0\})$ aS = Sa=S. (CO1)
  - (b) Show that in a periodic semi group every element has a power which is idempotent and hence show that in a periodic semi group there is at least one idempotent. (CO1)

## (OR)

- 2 (a) If S is a relation on a set X, then show that  $S^{\infty}$  is the smallest equivalence relation on X containing S. (CO1)
  - (b) If R is any binary relation on a set X, then prove that  $R^e = [R \cup R^{-1} \cup 1_{*}]^{\infty}$ . (CO1)

#### UNIT -II

- 3 (a) If R is a relation on a semi group S, then show that  $R^{\#} = (R^{c})^{e}$ . (CO2) (b) Show that for any two congruences  $\rho$  and  $\sigma$  on a group G,  $\rho \circ \sigma = \sigma \circ \rho$ . (CO2)
  - (OR)

4 (a)Show that a modular lattice is semi modular.

(b) Let A be a non empty set and let S be a semi group. If  $\varphi$ : A $\rightarrow$ S is an arbitrary mapping then show that there exists unique homomorphism  $\psi$  : F<sub>A</sub>  $\rightarrow$  such that  $\psi$  /A = $\varphi$ .(CO2)

#### **UNIT –III**

- 5 (a) Define a periodic semi group . If S is a periodic semi group then show that J = D. (CO3)
  - (b) If H is an H- class in a Semi group S then show that either  $H^2 \cap H = \phi$  or  $H^2 = H$  and H is a subgroup of S. (CO3)

## (OR)

#### (MA3T4B)

(CO2)

6 (a) If H and K are two group H – classes in the same D – class then show that H and K are isomorphic. (CO3)(b) State and prove Lallement's lemma. (CO3)

#### UNIT -IV

- 7 (a) Define 0- simple semi group. If M is a 0- minimal ideal of S then show that either  $M^2 = 0$ or M is a 0- simple semi group. (CO4)(b) If S is completely 0- simple, then show that S is regular. (CO4)

#### (OR)

- 8 (a) Show that a 0-Simple semi group is completely 0-Simple if and only if it contains a primitive idempotent. (CO4)
  - (b) If S is a regular semi group with zero in which every non zero idempotent is primitive, then show that S is a 0- direct union of completely 0- simple semi groups. (CO4)

#### UNIT -- V

- 9 (a) Show that N  $\rho$  is a normal sub group of G.
  - (b) If S is a finite congruence free semi group with out zero and if |S| > 2 then prove that S is a simple group. (CO5)

(OR)

10. If  $\rho$  and  $\sigma$  are proper congruences on  $S = \mu^0 [G: I, \land: \rho]$  then show that  $\rho \cap \sigma = [N_\rho \cap N_\sigma, \rho_1 \cap \sigma_1, \rho_\land \cap \sigma_\land]$  $\rho \lor \sigma = [N_\rho.N_\sigma, \rho_1 \lor \sigma_1, \rho_\land \lor \sigma_\land]$ 

(CO5)

(CO5)

# M.Sc. MATHEMATICS III SEMESTER MA3T4C: NUMBER THEORY

Subject Code :	MA3T4C	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives :** To develop problem solving skills and to acquire knowledge on basic concepts of Arithmetical Functions, Dirichlet Multiplication, Averages of Arithmetical Functions and Congruences.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Define and interpret the concepts of divisibility, congruence, Dirichlet product, multiplicative functions
CO2	Understand the concepts of averages of arithmetical functions, prove and apply properties of multiplicative functions such as the Euler phi function and of residues
CO3	Understand Chebyshev's functions $\psi(x)$ and $I(x)$ and the Relations connecting $I(x)$ and $\pi(x)$ , Some equivalent forms of the prime number theorem, Inequalities of $\pi(n)$ and pn, to study the applications of Shapiro's Tauberian theorem
CO4	Solve congruences of various types and use the theory of congruences in applications
CO5	Produce rigorous arguments on the number theory, such as in the use of Mathematical induction and well ordering principle in the proof of the theorems

#### **Mapping of Course Outcomes:**

	<b>PO1</b>	PO2	PO3	PO4	PO5
CO1			L3		
CO2	L4				
CO3			L3		
CO4				L4	
CO5					L3

## UNIT-1(18 Hours) (Online Mode)

**ARITHMETICAL FUNCTIONS AND DIRICHLET MULTIPLICATION:** Introduction, The Mobius function  $\mu(n)$ , The Euler Totient function  $\phi(n)$ , A relation connecting  $\phi$  and  $\mu$ , A product formula for  $\phi(n)$ , The Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, The Mangoldt function  $\Lambda(n)$ , Multiplicative functions, Multiplicative functions and Dirichlet multiplication, The inverse of a completely multiplicative function, Liouville's function  $\lambda(n)$ , The divisor function  $\sigma_z(n)$ . Generalised convolutions. (Sections 2.1 to 2.14 of Chapter 2)

#### UNIT-11(18 Hours)

AVERAGES OF ARITHMETICAL FUNCTIONS: Introduction, The big oh notation Asymptotic equality of functions, Euler's summation formula, Some elementary asymptotic formulas, The average order of d(n), The average order of divisor functions  $\sigma_z(n)$ , The average order of  $\varphi(n)$ , An application to

the distribution of lattice points visible from the origin, The average order of  $\mu(n)$  and  $\Lambda(n)$ , The partial sums of a Dirichlet product, Applications to  $\mu(n)$  and  $\Lambda(n)$ , Another identity for the partial sums of a Dirichlet product. (Sections 3.1 to 3.12 of Chapter 3)

UNIT-III(18 Hours)

SOME ELEMENTARY THEOREMS ON THE DISTRIBUTION OF PRIME NUMBERS: Introduction, Chebyshev's functions  $\psi(x)$  and I(x). Relations connecting I(x) and  $\pi(x)$ , Some equivalent forms of the prime number theorem, Inequalities of  $\pi(n)$  and  $p_n$ , Shapiro's Tauberian theorem, Application of Shapiro's theorem, An asymptotic formulae for the partial sums  $\sum_{\mu x} (1 / p)$ , The Partial Sums of the

Mobius function. (Sections 4.1 to 4.9 of Chapter 4)

#### UN1T-1V(18 Hours)

**CONGRUENCES:** Definition and basic properties of congruences, Residue classes and complete residue systems, Linear congruences, Reduced residue systems and Euler - Fermat theorem, Polynomial congruences modulo p, Lagrange's theorem, Applications of Lagrange's Theorem, Simultaneous linear congruences, The Chinese remainder theorem, Applications of the Chinese remainder theorem, Polynomial congruences with prime power moduli.

(Sections 5.1 to 5.9 of Chapter 5)

#### UN1T-V (18 Hours)

**FINITE ABELIAN GROUPS AND THEIR CHARACTERS:** Definitions, Examples of groups and subgroups, Elementary properties of groups, Construction of subgroups, Characters of finite abelian groups, The character group, The orthogonality relations for characters, Dirichlet characters, Sums involving Dirichlet characters.

(Sections 6.1 to 6.9 of Chapter 6)

**PRESCRIBED BOOK: "Introduction to Analytic Number Theory",** Tom M.Apostol, Narosa Publishing House, New Delhi.

**REFERENCE BOOK: "An Introduction to the Theory of Numbers",** Hardy G.H. and Wright E.M., Oxford Press.

(MA3T4C)

# M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Third Semester Mathematics Paper IV – NUMBER THEORY

Time: Three hours	Answer ONE question form each unit.	Maximum: 70
Marks		

## All questions carry equal marks.

#### UNIT –I

1 (a) Define the Dirichlet product. State and prove Mobius inversion formula (CO1)

(b) Show that if f and g are multiplicative, then so is their Dirichlet product. (CO1)

#### (OR)

2 (a) Let f be multiplicative. Then show that f is completely multiplicative if and only if  $f^{-1}(n) = \mu(n)f(n)$  (CO1)

(b) If  $\alpha$  has a Dirichlet inverse then show that the equation

$$G(\mathbf{x}) = \sum_{n \le x} \alpha(n) F\left(\frac{x}{n}\right) \text{ implies } F(x) = \sum_{n \le x} \alpha^{-1}(n) G\left(\frac{x}{n}\right)$$
(CO1)

#### UNIT –II

(b) For all  $x \ge 1$ , show that  $\sum_{n \le x} d(n) = x \log x + (2C - 1)x + O(\sqrt{x})$ , where C is Euler's constant.

(CO2)

(CO2)

(CO2)

- (OR)
- 4 (a) State and prove Legendre's identity.

(b) If a and b are positive real numbers such that ab = x, then show that

$$\sum_{\substack{q,d\\qd\leq x}} f(d)g(q) = \sum_{n\leq a} f(n)G\left(\frac{x}{n}\right) + \sum_{n\leq b} g(n)F\left(\frac{x}{n}\right) - F(a)G(b)$$
(CO2)

#### UNIT –III

5(a) Define Chebyshev function. State and prove Abel's identity. (CO3)(b) Show that the following identity's are logically equivalent.

$$\lim_{x \to \infty} \frac{\pi(x)\log x}{x} = 1 \qquad \qquad \lim_{x \to \infty} \frac{\Im(x)}{x} = 1 \qquad \qquad \lim_{x \to \infty} \frac{\psi(x)}{x} = 1 \qquad (CO3)$$
(OR)

6 (a) Show that for every integer  $n \ge 2$ ,

$$\frac{1}{6}\frac{n}{\log n} < \pi(n) < 6\frac{n}{\log n} \tag{CO3}$$

(b) Show that the prime number theorem implies  $\lim_{x\to\infty} \frac{M(x)}{x} = 0$ (CO3)

# UNIT –IV

UNII -IV	
7 (a) Assume (a, m) =1. Then show that the linear congruence ax $\equiv b \pmod{m}$ has exact	ly one
solution.	(CO4)
(b) State and prove Euler – Fermat theorem.	(CO4)
(OR)	
8 (a) State and prove Wilson's theorem.	(CO4)
(b) State and prove Chinese remainder theorem.	(CO4)
UNIT –V	
9 (a) If G is finite and a $\in$ G, then show that there is an positive integer n $\leq$ $ G $ such that a	$^{n}=e.$
	(CO5)
(b) Show that a finite abelian group G of order n has exactly n distinct characters.	(CO5)
(OR)	

10 (a) Let A\* denote the conjugate transpose of a matrix A. Then show that AA\*= Ni, where I is the nxn identity matrix and hence  $n^{-1}A$  is the inverse of A. (CO5)

(b) If  $\chi$  is any non principal character mod k and if  $x \ge 1$  then show that

$$\sum_{n \le x} \frac{\chi(n)}{n} = \sum_{n=1}^{\infty} \frac{\chi(n)}{n} + O\left(\frac{1}{x}\right)$$

$$\sum_{n \le x} \frac{\chi(n) \log n}{n} = \sum_{n=1}^{\infty} \frac{\chi(n) \log n}{n} + O\left(\frac{\log x}{x}\right)$$
$$\sum_{n \le x} \frac{\chi(n)}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{\chi(n)}{\sqrt{n}} + O\left(\frac{1}{\sqrt{x}}\right)$$

(CO5)

# M.Sc. MATHEMATICS III SEMESTER MA3T5A: OPERATIONS RESEARCH-I

Subject Code :	MA3T5A	I A Marks	30		
No. of Lecture Hours / Week	06	End Exam Marks	70		
Total Number of Lecture Hours	90	Total Marks	100		
Seminar	01	Exam Hours	03		

**Objectives :** To develop problem solving skills and to acquire knowledge on basic concepts of in linear programming problems, Transportation problems, Assignment problems and Job sequencing.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Formulate and solve a linear programming problem
CO2	Convert standard business problems into linear programming problems and can solve using simplex algorithm
CO3	Formulate and solve transportation problems
CO4	Formulate and solve the Assignment problem and can compare Transportation and Assignment problem
CO5	Formulate and solve Job sequencing problems

# Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1				L3	
CO2				L4	
CO3				L5	
CO4					L4
CO1 CO2 CO3 CO4 CO5					L5

# UNIT – I (18 Hours) (Online Mode)

**Mathematical Background :** Lines and hyperplanes: Convex sets, Convex sets and hyperplanes, Convex cones. [Sections 2.19 to 2.22 of Chapter 2of [1]].

**Theory of the simplex method :** Restatement of the problem, Slack and surplus Variables, Reduction of any feasible solution to a basic feasible solution, Some definitions and notations, Improving a basic feasible solution, Unbounded solutions, Optimality conditions, Alternative optima, Extreme points and basic feasible solutions.

[Sections 3.1, 3.2, 3.4 to 3.10 of Chapter 3 of [1]]

# UNIT –II(18 Hours)

**Detailed development and Computational aspects of the simplex method**: The Simplex method, Selection of the vector to enter the basis, degeneracy and breaking ties, Further development of the transportation formulas, The initial basic feasible solution –artificial variables, Tableau format for simplex computations, Use of the tableau format, conversion of a minimization problem to a maximization problem, Review of the simplex method, Illustrative examples. [Sections 4.1 to 4.5, 4.7 to 4.11 of Chapter 4 of [1]].

# UNIT –III (18 Hours)

**Transportation problem:** Introduction, properties of the matrix **A**, The Simplex Method and transportation problems, Simplifications resulting from all  $y_{ij}^{\alpha\beta} = \pm 1$  or 0, The Transportation Problem Tableau, Bases in the transportation Tableau, The Stepping-Stone algorithm, Determination of an initial basic feasible solution, Alternative procedure for computing  $z_{ij}$  – $c_{ij}$ ; duality. [Sections 9.1 to 9.7 & 9.10, 9.11 of Chapter 9 of [1]].

# UNIT –IV (18 Hours)

**The Assignment problem :** Introduction, Description and Mathematical statement of the problem, Solution using the Hungarian method, The relationship between Transportation and Assignment problems, Further treatment of the Assignment problem, The Bottleneck Assignment problem. (Chapter 6 of [2])

# UNIT V (18 Hours)

**Job Sequencing:** Introduction, Classification, Notations and Terminologies, Assumptions, Sequencing Problems: Sequence for n jobs through two machines, Sequence for n jobs through three machines, Sequence for 2 jobs through m machines, Sequence for n jobs through m machines (Sections 12.1 to 12.5 of chapter 12 of [3])

# **PRESCRIBED BOOKS:**

- [1] "Linear programming", G.Hadley, Addison Wesley Publishing Company.
- [2] " **Introduction to Mathematical Programming**", Benjamin Lev and Howard J. Weiss, Edward Arnold Pub, London, 1982.
- [3] "Operations Research- Algorithms and Applications", Rathindra p. Sen, PHI

# **REFERENCE BOOK: "Operations Research",** Nita H.Shah, Ravi M. Gor, Hardik Soni, PHI

# **M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Third Semester Mathematics** Paper V – Operations Research-I

#### **Time: Three hours** Answer ONE question form each unit. Max: 70 Marks All questions carry equal marks.

## UNIT – I

- 1.(a) Define convex set. Prove that the collection of all feasible solutions to a L.P.P constitutes a convex set whose extreme points correspond to the B.F.S. (CO1) (CO1)
  - (b) Find all basic feasible solutions for the system
    - $2x_1 + 6x_2 + 2x_3 + x_4 = 3$  $6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$  $X_i \ge 0, i=1,2,3,4.$
- (**OR**)
- 2. (a) Prove that the set of all convex combinations of a finite number of points  $x_1, x_2, \dots, x_n$  is a convex set. (CO1)
  - (b) Define an extreme point of a convex set. If a closed strictly bounded convex set X has a finite number of extreme points, then prove that any point in the set can be written as a convex combination of the extreme points. (CO1)

(**OR**)

## 3. Write the simplex algorithm.

(CO2)

- 4. Solve the following L.P.P using simplex method. (CO2)max  $z = 6x_1 - 2x_2$ 
  - $sub: 2x_1 x_2 \le 2$  $x_1 \leq 4$  $x_1, x_2 \ge 0.$

## UNIT – III

5. (a) Explain the procedure of obtaining the optimum solution to the transportation problem. (b) Find the optimal solution by finding the IBFS using the Vogel's method for the following.

	Ι	II	III	Ava
А	2	7	4	5
В	3	3	1	8
С	5	4	7	7
D	1	6	2	14

Req	7	9	18	
:				

(CO3)

(CO4)

# (OR)

6. Solve the following Transportation Problem using stepping stone algorithm.

	Ι	Π	III	IV	supply
А	40	44	48	35	160
В	37	45	50	52	150
С	35	40	45	50	190
Demand	80	90	110	220	

## UNIT - IV

7.(a) Explain the Hungarian method.

(b) Solve the following assignment problem by using Hungarian method. (CO4)

	Ι	II	III	IV	V
А	45	30	65	40	55
В	50	30	25	60	30
С	25	20	15	20	40
D	35	25	30	30	20
Е	80	60	60	70	50

# (OR)

8. Formulate the assignment problem mathematically and solve the following by using bottle neck assignment algorithm. (CO4)

	Α	В	С	D
1	2	4	2	4
2	8	5	4	5
3	4	6	8	9
4	8	4	2	4

UNIT - V

- 9 (a) Discuss Johnson's procedure for determining an optimal sequence for processing n jobs through two machines . (CO5)
  - (b) Five jobs are performed first on machine X and then on machine Y. Then time taken in hours by each job on each machine is given below: (CO5)

Jobs	А	В	С	D	Е
Time on machine X	12	4	20	14	22
Time on machine Y	6	14	16	18	10

Determine the optimum sequence of jobs that minimizes the total elapsed time to complete the jobs. Also compute the idle time.

(OR)
------

10 (a) Explain how to process n jobs through m machines.

(CO5)

(b) Find the optimal sequence for the following problem to minimize time and also obtain elapsed time: (CO5)

Jobs	Machine A	Machine B	Machine C
1	13	8	13
2	8	9	12
3	12	10	11
4	7	7	14
5	10	6	15
6	6	11	14

# M.Sc. MATHEMATICS III SEMESTER MA3T5B: THEORY OF COMPUTER SCIENCE-I

Subject Code :	MA3T5B	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives :** To acquire knowledge on basic concepts of Theory of Automata, Formal Languages, Regular sets and Regular Grammars, Context-free Languages, Push down Automata.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:				
CO1	Interpret Mealy and Moore Models and understand Minimization of Finite automata				
CO2	Distinguish different computing languages and classify their respective types and recognize and comprehend formal reasoning about languages				
CO3	Apply Pumping Lemma, understand the Closure Properties of Regular Sets study Regular Sets and Regular Grammars				
CO4	Understand pushdown Automata and Context-free languages				
CO5	Understand Context-free Languages				

# Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1				L4	
CO2	L3				
CO3				L4	
<b>CO4</b>	L3				
CO5	L3				

# UNIT-I (18 Hours) (Online Mode)

**Mathematical Preliminaries**: Sets, Relations and Functions, 1.2 Graphs and Trees, 1.3 Strings and their properties, 1.4 Principle of Induction

**The Theory of Automata:** 2.1Definition of an Automation, 2.2 Description of a Finite Automation 2.3 Transition Systems, 2.4 Properties of Transition Functions

2.5 Acceptability of a String by a Finite Automation, 2.6 Nondeterministic Finite State Machines, 2.7 The Equivalence of DFA and NDFA, 2.8 Mealy and Moore Models,
2.9 Minimization of Finite Automata (Chapters 1 and 2)

# UNIT-II (18 Hours)

**Formal Languages** : 3.1 Basic Definitions and Examples 3.2 Chomsky Classification of Languages 3.3 Languages and their relation 3.4 Recursive and Recursively Enumerable Sets 3.5 Operations on Languages, 3.6 Languages and Automata (Ch. 3)

# UNIT-III (18 Hours) Regular Sets and Regular Grammars :

4.1 Regular Expressions 4.2 Finite Automata and Regular Expressions
4.3 Pumping Lemma for Regular Sets 4.4 Application of Pumping Lemma
4.5 Closure Properties of Regular Sets 4.6 Regular Sets and Regular Grammars (Chapter 4)

# UNIT-IV (18 Hours)

**Context-free Languages** : 5.1 Context-free Languages and Derivation Trees, 5.2 Ambiguity in Context-free Grammars ,5.3 Simplification of Context-free grammars, 5.4 Normal Forms for Context-free grammars, 5.5 Pumping lemma for Context-free Languages, 5.6 Decision Algorithms for Context-free Languages (Chapter 5)

# UNIT-V (18 Hours)

**Pushdown Automata: 6.1** Basic Definitions, 6.2Acceptance by pda, 6.3Pushdown Automata and Context-free languages, 6.4Parsing and Pushdown automata(Chapter 6)

**PRESCRIBED BOOK:** KLP Mishra & N.Chandrasekharan, **Theory of Computer Science** (Automata,Languages and Computation), Prentice Hall of India.

**REFERENCE BOOKS : 1. "Introduction to Automata Theory"**, Hopcroft J.E & Ullman J.D., **Languages and Computation**, Narosa Publishing House, 1987.

2. "Introductory Theory of Computer Science", E.V. Krishna Murthy, Affiliated East – West Press., New Delhi, 1984.

# M.Sc. DEGREE EXAMINATION MODEL QUESTION PAPER Third Semester Mathematics Paper V- THEORY OF COMPUTER SCIENCE - I

# Time: Three hoursAnswer ONE question form each unit.Maximum: 70MAll questions carry equal marks.

## UNIT I

(a) Prove that a tree with n vertices has n-1 edges.
(b) Prove that for every NDFA there exists a DFA which simulates the behavior of NDFA. Alternatively if N is the set accepted by NDFA, then there exists a DFA which also accepts L.
(CO1)

#### (OR)

- 2. (a) Define a binary tree. Prove that the number of vertices in a binary tree is odd. (CO1)
  - (b) Construct a deterministic automation equivalent to  $M = (\{q_0 q_1\}, \{0, 1\}, \delta, q_0, \{q_0\}),$ where  $\delta$  is given by

State/ ∑	0	1
$\rightarrow q_0$	$q_0$	$\mathbf{q}_1$
<b>q</b> 1	<b>q</b> 1	$q_0, q_1$

1.

(CO1)

#### **UNIT II**

3. (a) Prove that a context sensitive language is recursive. (CO2) (b) Show that each of the classes  $L_0$ ,  $L_{csl}$ ,  $L_{rdl}$ , is closed under concatenation. (CO2)

#### (OR)

4 (a) Find a grammer generating L = {a<sup>n</sup> b<sup>n</sup> c<sup>i</sup> / n≥1, i≥0 }. (CO2)
(b) Show that there exists a recursive set which is not context sensitive language over {0,1}

#### **UNIT III**

- 5 (a) Prove that the language of any DFA can be represented by a regular expression. (CO3) (b) Prove that  $(a+b)^* = a^*(ba^*)^*$ . (CO3)
  - (OR)

6 (a)State and prove pumping lemma for regular sets.(CO3)(b)Show that  $L = \{ww/w \in \{a,b\}^*\}$  is not regular.(CO3)

## UNIT IV

- 7 (a) Define Chomsky normal form. Obtain a grammar in Chomsky normal form equivalent to the grammar :  $S \rightarrow aAbB, A \rightarrow aA|a, B \rightarrow bB|b$ . (CO4)
- (b) When do we say that a context free grammar is ambiguous? Show that the grammar  $S \rightarrow a|abSb|aAb, A \rightarrow bS/aAAb$  is ambiguous. (CO4)

#### (OR)

- 8 (a) Construct a reduced grammer equivalent to the grammar :  $S \rightarrow aAa, A \rightarrow sb | bc, c | DaA, C \rightarrow abb | DD, E \rightarrow aC, D \rightarrow aDA |$ . (CO4)
  - (b) Show that  $L = \{a^n b^n c^n / n \ge 1\}$  is not context free but context sensitive. (CO4)

#### UNIT V

- 9 (a) If A is a pda (push down automation), prove that there exists a context free grammar such that L(G) = N(A). (CO5)
  - (b) Construct a pda accepting  $\{a^n b^m a^n / m, n \ge 1\}$  by null store. Also construct the corresponding context free grammar accepting the same set. (CO5)

## (OR)

- 10 (a) If L is a context free language, then prove that there exists a pda A accepting L by empty store ie., L=N(A). (CO5)
  - (b) Construct a pda A equivalent to the following context free grammar :  $S \rightarrow OBB, B \rightarrow OS|IS|O$ .

(CO5)

# M.Sc. MATHEMATICS IV SEMESTER MA4T1: NON COMMUTATIVE RINGS

Subject Code :	MA4T1	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives:** To acquire knowledge on advanced concepts of non commutative rings i.e., radical theory, prime and primitive rings, completely reducible rings and modules, Injective and Projective modules.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:					
CO1	Characterize primitive rings and completely reducible modules.					
CO2	Decide whether a given ring or module, or a class of rings or modules, is Noetherian /artinian/semisimple, by applying the characterizations discussed in the course.					
CO3	Identify local rings, semi-perfect rings					
CO4	Characterize Injective and Projective modules					
CO5	Analyze complete ring of quotients and rings of endomorphisms of injective modules					

# **Mapping of Course Outcomes:**

	PO1	PO2	PO3	PO4	PO5	PO6	<b>PO7</b>
CO1	Н						
CO2							Н
CO3	Н						
<b>CO4</b>							Н
CO5	Μ						

## UNIT I

**Classical theory of Associative rings :** Primitive Rings, Radicals, Completely reducible modules. [Sections 3.1, 3.2, 3.3 of Chapter 3]

# UNIT II

**Classical theory of Associative rings:** Completely reducible rings, Artinian and Noetherian Rings [Sections 3.4, 3.5 of Chapter 3]

# UNIT III

**Classical theory of Associative rings:** On lifting idempotents, Local and Semi perfect rings. [Sections 3.6, 3.7 of Chapter 3]

## UNIT IV Injectivity and Related concepts: Projective modules, Injective modules. [Sections 4.1, 4.2 of Chapter 4]

# UNIT V

**Injectivity and Related concepts:** The complete ring of quotients, Rings of endomorphisms of Injective modules. [Sections 4.3, 4.4 of Chapter 4]

**PRESCRIBED BOOK:** J.Lambek, *Lectures on Rings and Modules*, Blaisdell Publications(2009).

**REFERENCE BOOK:** Thomas W. Hungerford , *Algebra*, Springer publications(1974).

## M.Sc. MATHEMATICS IV SEMESTER MA4T2: MEASURE AND INTEGRATION

Subject Code :	MA4T2	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives:** To develop skills and to acquire knowledge on basic concepts of Lebesgue Measure, Lebesgue Integral, Measurable Functions, Outer Measure and Product Measures.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Understand the concept of measure and properties of Lebesgue measure
CO2	Study the properties of lebesgue integral and compare it with Riemann integral.
CO3	Find the derivative of an Integral and Integral of a derivative for the functions of bounded variation.
CO4	Construct different measures for P(X) and study their properties
CO5	Define product measure and study the concept of integral with respect to product measure.

### Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5	PO6	<b>PO7</b>
CO1	Н						
CO2							Н
CO3	Н						
CO4							Η
CO5	Н						

# UNIT-I

**Lebesgue Measure**: Introduction, Outer measure , Measurable sets and Lebesgue measure, A non measurable set, Measurable functions, Littlewood's three principles (Chapter 3)

## UNIT-II

**The Lebesgue Integral**: The Riemann Integral, The Lebesgue Integral of a bounded function over a set of finite measure, The Integral of a non- negative function, The general Lebesgue Integral. (Sections 4.1 to 4.4 of Chapter 4).

# **UNIT-III**

**Differentiation and Integration**: Differentiation of monotone functions, Functions of bounded variation, Differentiation of an Integral, Absolute continuity. (Sections 5.1 to 5.4 of Chapter 5)

# UNIT-IV

**Measure and Integration**: Measure spaces, Measurable functions, Integration, General Convergence theorems, Signed Measures, The Radon-Nikodym theorem. (Sections 11.1 to 11.6 of Chapter 11)

## **UNIT-V**

**Measure and Outer Measure**: Outer Measure and Measurability, The Extension theorem, Product measures. (Sections 12.1, 12.2 & 12.4 of Chapter 12).

PRESCRIBED BOOK: H.L.Royden, Real Analysis, Third Edition, Pearson Pub(1988).

#### **REFERENCE BOOKS** :

[1] P.R.Halmos, Measure Theory, Springer-Verlag, (1974).

[2] V.I. Bogachev, Measure Theory, Springer -Verlag, (1997).

# M.Sc. MATHEMATICS IV SEMESTER MA4T3: FUNCTIONAL ANALYSIS-II

Subject Code :	MA4T3	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives:** To develop skills and to acquire knowledge on advanced concepts in Category theorem, Open mapping theorem, Closed Graph theorem, Banach's theorem and it's applications, Spectral theory etc.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:							
CO1 Characterize the category of normed spaces using Category theorem and differentiate								
COI	pointwise convergence of linear operators							
CO2	Apply Banach's Theorem to Linear Equations, differential Equations and Integral Equations.							
CO3	Demonstrate Spectral properties of Bounded Linear Operators							
CO4	Understand Banach algebras, Demonstrate spectral properties of compact linear operators							
CO5	Study Operator equations involving Compact linear operators							

#### Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5	PO6	PO7
CO1	Н						
CO2							Н
CO3	Н						
CO4	Μ						
CO5							Н

### UNIT- I

Category Theorem, Uniform Boundedness Theorem, Strong and Weak Convergence, Convergence of Sequences of Operators and Functionals, Open Mapping Theorem, Closed Linear Operators, Closed Graph Theorem.

(Sections 4.7, 4.8, 4.9, 4.12 and 4.13 of Chapter 4).

#### UNIT-II

Banach Fixed Point Theorem, Application of Banach's Theorem to Linear Equations, Application of Banach's theorem to differential Equations, Application of Banach's theorem to Integral Equations. (Chapter 5)

#### UNIT –III

Spectral theory in finite dimensional normed spaces, Basic concepts, Spectral properties of Bounded Linear Operators, Further properties of resolvent and Spectrum. (Sections 7.1 to 7.4 of Chapter -7)

### UNIT –IV

Banach Algebras, Further properties of Banach Algebras, Compact linear operators on Normed spaces, Further properties of compact linear operators, Spectral properties of Compact linear operators on Normed spaces. (Sections 7.6, 7.7 of Ch. 7 & Sections 8.1 to 8.3 of Ch.8)

### UNIT –V

Further Spectral properties of Compact linear operators, Operator equations involving Compact linear operators, Further Theorems of Fredholm type, Fredholm alternative. (Sections 8.4 to 8.7 of Chapter -8)

**PRESCRIBED BOOK:** Erwin Kreyszig, *Introductory Functional analysis with Applications*, John Wiley & Sons(1978).

**REFERENCE BOOK:** M. Thamban Nair, *Functional Analysis- A First Course*, PHI(2002).

## M.Sc. MATHEMATICS IV SEMESTER MA4T4A: ALGEBRAIC CODING THEORY

Subject Code :	MA4T4A	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives:** To acquire knowledge on basic concepts of linear codes, parity-check matrices, Gilbert bound, Hamming bound, Singleton bound, Cyclic Linear Codes, Perfect Codes etc.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Understand the basic concepts of coding theory and demonstrate encoding and decoding using MLD
CO2	Demonstrate the concepts of error correction and detection, understand linear codes, calculate a basis for linear codes and its dual.
CO3	Calculate generator matrix, parity check matrix for linear codes and its dual using algorithms and decoding using CMLD and IMLD.
CO4	Understand perfect codes by illustrating with examples of Hamming codes and calculating Hamming bound, Gilbert Varshamov Bound, decoding with Reed-Muller codes.
CO5	Study cyclic linear codes and dual cyclic codes and construct cyclic linear codes of a given length.

### Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5	PO6	PO7
CO1	Μ						
CO2							Н
CO3	Μ						
CO4							Н
CO5	Н						

### UNIT –I

**Introduction to Coding Theory:** Introduction, Basic assumptions, Correcting and Detecting error patterns, Information Rate, The Effects of error Correction and Detection, Finding the most likely codeword transmitted, Some basic algebra, Weight and Distance, Maximum likelihood decoding, Reliability of MLD. (Section 1.1 to 1.10 of Chapter 1)

### UNIT – II

Introduction to Coding Theory : Error- detecting Codes, Error – correcting Codes.

**Linear Codes :** Linear Codes, Two important subspaces, Independence, Basis, Dimension, Matrices, Bases for C=  $\langle S \rangle$  and C<sup>⊥</sup> (Sec. 1.11, 1.12 of ch.1 & Sec. 2.1 to 2.5 of ch.2).

### UNIT – III

**Linear Codes :** Generating Matrices and Encoding , Parity – Check Matrices, Equivalent Codes, Distance of a Linear Code, Cosets, MLD for Linear Codes, Reliability of IMLD for Linear Codes.(section 2.6 to 2.12 of chapter 2)

### UNIT –IV

**Perfect and Related Codes:** Some bounds for Code, Perfect Codes, Hamming Codes, Extended Codes, The extended Golay Code, Decoding the extended Golay Code, The Golay code, Reed – Mullar Codes, Fast decoding for RM (1,m).(Chapter 3)

#### UNIT –V

**Cyclic Linear Codes :** Polynomials and Words, Introduction to Cyclic codes, Polynomials encoding and decoding, Finding Cyclic Codes, Dual Cyclic Codes.(Chapter 4)

**PRESCRIBED BOOK:** D.G. Hoffman, D.A. Lanonard , C.C. Lindner, K. T. Phelps, C. A. Rodger, J.R.Wall, *Coding Theory- The Essentials*", Marcel Dekker(1991).

**REFERENCE BOOK**: J.H. Van Lint, Introduction to coding Theory, Springer Verlag(2013).

## M.Sc. MATHEMATICS IV SEMESTER MA4T5A: OPERATIONS RESEARCH-II

Subject Code :	MA4T5A	I A Marks	30
No. of Lecture Hours / Week	06	End Exam Marks	70
Total Number of Lecture Hours	90	Total Marks	100
Seminar	01	Exam Hours	03

**Objectives:** To develop problem solving skills of linear programming problems using Two-Phase method, Duality theory, Revised Simplex method, Game theory and Dynamic Programming.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	Solve a linear programming problem using Two phase Simplex Method
CO2	Find the dual of a linear programming problem and solve the Problem
CO3	Solve a linear programming problem using Revised Simplex Method
CO4	Solve integer programming problems and game theory problems.
CO5	Formulate and solve dynamic programming problems related to the society and industry.

#### Mapping of Course Outcomes:

	<b>PO1</b>	PO2	PO3	PO4	PO5	PO6	<b>PO7</b>
CO1	Н						
CO2							Μ
CO3	Μ						
CO4						Η	
CO5			Н				

### UNIT –I

**Further Discussion of the Simplex Method:** The two phase Method for Artificial variables, phase-I, Phase-II, Numerical examples of the two phase method. (Sections 5.1 to 5.4 of Chapter -5 of [1]]

### UNIT –II

**Duality theory and its Ramifications:** Alternative formulations of linear programming problems, Dual linear programming problems, Fundamental properties of dual problems, Other formulations of dual problems, Unbounded solution in the primal, The dual simplex algorithm –an example. Post optimality problems, Changing the price vector, Changing the requirements vector, Adding variables or constraints (Sections 8.1 to 8.7 & 8.10 of Chapter 8 of [1]).

## UNIT –III

**The Revised simplex method:** Introduction, Revised simplex method: Standard form I, Computational procedure for standard form I, Revised simplex method: Standard form II, Computational procedure for standard form II, Initial identity matrix for phase–I, Comparison of the simplex method and Revised simplex method.

(Sections 7.1 to 7.6 & 7.8 of Chapter 7 of [1]).

## UNIT –IV

**Game theory:** Game theory and Linear programming, Introduction, Reduction of a game to a linear programming problem, Conversion of a linear programming problem to a game problem. (Sections 11.12 to 11.14 of Chapter 11 of [1] and Section 10.3 of Chapter10 of [2]) **Integer programming:** Introduction, Gomory's cut, Balas Implicit Enumeration technique. (Sections 7.1, 7.2 and 7.4 of Chapter 7 of [2]).

## UNIT V

**Dynamic Programming:** Introduction, Characteristics of Dynamic Programming problem, Deterministic Dynamic Programming: Dynamic Programming approach to Shortest Route Problem, Dynamic Programming approach to Resource Allocation: Equipment, Replacement, Cargo loading, and capital budgeting. Dynamic Programming approach to linear programming, Stochastic Dynamic Programming. (Sections 6.1 to 6.4 of chapter 6 of [3])

PRESCRIBED BOOKS:

- [1] G.Hadley, Linear programming, Addison Wesley Publishing Company(1978).
- [2] Benjamin Lev and Howard J. Weiss, *Introduction to Mathematical Programming*, Edward Arnold Pub, London(1982).
- [3] Rathindra p. Sen, Operations Research-Algorithms and Applications, PHI(2009).

**REFERENCE BOOK:** Nita H.Shah, Ravi M. Gor, Hardik Soni, *Operations Research, PHI*(2010).



**P.B SIDDHARTHA COLLEGE OF ARTS & SCIENCE, VIJAYAWADA-10.** (An Autonomous College in the jurisdiction of Krishna University, Machilipatnam)

# ADD-ON COURSE

## TITLE OF THE PAPER: MATHEMATICAL MODELLING

#### **UNIT-I: Basic Concepts**

- Introduction.
- Definition and Terminology.
- Linear and Nonlinear Differential Equations.
- Solution of a Differential Equation.
- Origins and Formation of a Differential Equations.
- Differential Equation of a Family of Curves.
- Physical Origins of Differential Equations.
- General, Particular and Singular Solutions.

#### UNIT-II: Mathematical Modelling through Ordinary Differential Equations of First Order

- Growth and Decay.
- Dynamics of Tumour Growth.
- Radioactivity and Carbon Dating.
- Compound Interest.
- Belt or Cable friction.
- Temperature Rate of Change (Newton's Law of Cooling).
- Diffusion.
- Biological Growth.
- A Problem in Epidemiology.
- The Spread of Technological Innovations.
- Mixture Problems.
- Absorption of Drugs in Organs or Cells.
- Rate of Dissolution.
- Chemical Reactions Law of Mass Action.
- One dimensional Heat Flow.
- Application in Economics.
- Orthogonal Trajectories.

#### UNIT-III: Mathematical Modelling through System of Linear Differential Equations

- Definitions and Solutions.
- Solution of a system of Linear Equations with Constant Coefficients.
- An Equivalent Triangular System.
- Degenerate Case.
- Central Force System, Newton's Law of Gravitation: Kepler's Laws of Planetary Motion.
- Motion of a Particle in the Gravitational Field of Earth: Satellite Motion.
- Vibration of a Coupled System.
- Compartment Systems.

- Mixture problem.
- Concentration of a drug in a Two-compartment system
- Some Further Applications.

Prescribed Text book:				
S.No	AUTHOR	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1.	Zafar Ahsan	Differential Equations and their Applications. Second edition	PHI Learning Private Limited, New Delhi	2010

 Unit: 1

 Chapter 1: Sec 1.1, 1.2, 1.3, 1.4, 1.5, 1.6 of Prescribed Text book

 Unit: 2

#### Chapter4:

Sec 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12, 4.13, 4.14, 4.15, 4.17 & 4.20 of Prescribed Text book

<u>Unit: 3</u>

Chapter 7: Sec 7.1, 7.2, 7.3, 7.4, 7.6, 7.7, 7.8, 7.10, 7.14 of Prescribed Text book

# DEPARTMENT OF MATHEMATICS ADD-ON COURSE ADVANCED MATHEMATICAL TECHNIQUES

# UNIT-I:(15Hours)

APPLICATIONS OF HIGHER ORDER DIFFERENTIAL EQUATIONS: Rectilinear motion (simple harmonic motion), The Simple Pendulum, Damped Motion, Forced Motion, Detection of Diabetes. (Sections 6.1 to 6.4 and 6.12 of Chapter 6)

## UNIT-II(15Hours):

LAPLACE TRANSFORMS AND THEIR APPLICATIONS TO DIFFERENTIAL EQUATIONS: Introduction, Properties of Laplace Transform, Unit Step Functions, Unit Impulse Functions, Solution of a Linear Differential Equation with Constant Coefficients Using Transform Methods, Applications of Laplace Transforms.

(Sections 8.1 to 8.5 and 8.6.1, 8.6.2 of Chapter 8).

## UNIT-III(15Hours):

CALCULUS OF VARIATIONS AND ITS APPLICATIONS: Introduction, The Variation of Functional and Euler's Equations, Functionals depending on n Unknown Functions, Functionals depending on Higher-order Derivatives.

(Sections 10.1 to 10.4 of Chapter 10).

### Prescribed Text Book:

"Differential Equations and Their Applications" Second Edition, Zafar Ahsan, PHI Publishers.