22CA1T3: MATHEMATICAL AND STATISTICAL FOUNDATIONS

Course Name	Mathematical and Statistical Foundations		L	T	P	C	CIA	SEE	TM
Course Code	22CA1T3		4	0	0	4	30	70	100
Year of Introduction: 2022	Year of Offering: 2022	Year of Revision: 2022	Percentage of Revision: Nil						
L-Lecture, T-Tutorial, **P**-Practical, **C**-Credits, **CIA**-Internal Marks, **SEE**-External Marks, **TM**-Total Marks									

Course Description and Purpose:
This course helps in learning Mathematical and Statistical foundations to understand the basics and apply them in *Computer Science*, *Data Science* and *Analytical Applications*.

Course Objectives:
To understand and apply Mathematical and Statistical Foundations including *Recursion*, *Advanced Counting Techniques*, *Relations*, *Graphs*, *Probability Laws*, *Discrete Distributions*, *Inferences on the Mean and the Variance of a Distribution* and *Inferences on Proportions*.

Specific objectives include:
- ✓ To understand *Mathematical Foundations* and *Recursion*.
- ✓ To learn and apply *Advanced Counting Techniques*.
- ✓ To understand the *Relations* and *Applications of Graphs*.
- ✓ To learn and apply *Probability Laws* and *Discrete Distributions*.
- ✓ To understand *Inferences on the Mean and the Variance of a Distribution* and *Inferences on Proportions.*

Course Learning Outcomes:
Upon successful completion of the course, the student will be able to:
CO1: Understand *Mathematical Foundations* and *Recursion*.
CO2: Learn and apply *Advanced Counting Techniques*.
CO3: Understand the *Relations* and *Applications of Graphs*.
CO4: Learn and apply *Probability Laws* and *Discrete Distributions*.
CO5: Understand *Inferences on the Mean and the Variance of a Distribution* and *Inferences on Proportions.*

Course Content:

UNIT I (12 Hours)

The Foundations: Logic and Proofs: Propositional Logic, Propositional Equivalences, Predicates and Quantifiers, Nested Quantifiers.
Introduction and Recursion: Mathematical Induction, Strong Induction and Well-Ordering, Recursive Definitions and Structural Induction, Recursive Algorithms, Program Correctness.

UNIT II (12 Hours)

Advanced Counting Techniques: Recurrence Relations, Solving Linear Recurrence Relations, Divide and Conquer Algorithms and Recurrence Relations, Generating Functions, Inclusion & Exclusion, Applications of Inclusion & Exclusion.

UNIT III (12 Hours)

Relations: Relations and Their Properties, Equivalence Relations, Partial Orderings.
Graphs: Graphs and Graph Models, Graph Terminology and Special Types of Graphs, Representing Graphs and Graph Isomorphism's, Connectivity, Euler and Hamilton Paths, Shortest Path Problems, Planar Graphs, Graph Coloring.

UNIT IV (12 Hours)

Some Probability Laws: Axioms of Probability, Conditional Probability, Independence of the Multiplication Rule, Bayes' Theorem.

Discrete Distributions: Random Variables, Discrete Probability Densities, Expectation and Distribution Parameters, Binomial Distribution, Poisson Distribution, Simulating a Discrete Distribution.

UNIT V (12 Hours)

Inferences on the Mean and the Variance of a Distribution: Hypothesis Testing, Significance Testing, Hypothesis and Significance Test on the Mean, Hypothesis Tests on the Variance.

Inferences on Proportions: Estimating Proportions, Testing Hypothesis on a Proportion, Comparing two Proportions: Estimation, Comparing two Proportions: Hypothesis Testing.

Reference Textbooks:

1. Susan Milton and Jesse C. Arno ld, Introduction to Probability and Statistics, Fourth Edition, November 2002.
2. William Mendenhall, Robert J Beaver, Barbara M Beaver, Introduction to Probability and Statistics, Twelth Edition, Thomson, January 2012.
3. Kenneth H Rosen, Discrete Mathematics and its Applications, 6th Edition, McGraw-Hill, Chapters [1-10], 2007.
4. Ralph P. Grimaldi, B.V. Ramana, Discrete and Combinational Mathematics, 5th Edition, Pearson Education, 2008.
5. Swapan Kumar Sarkar, A Text Book of Discrete Mathematics, S.Chand, 2008.
6. D.S.Malik andM.K.Sen, Discrete Mathematical Structures, Thomson, 2006.

PARVATHANENI BRAHMAYYA SIDDHARTHA COLLEGE OF ARTS & SCIENCE

(An Autonomous College in the jurisdiction of Krishna University)

M.C.A, First Semester

Course Name: Mathematical and Statistical Foundations

Course Code: 22CA1T3

(w.e.f admitted batch 2022-23)

Time: 3 Hours **Max Marks: 70**

SECTION-A

Answer all questions: **5 × 4 = 20 Marks**

1. (a) Show that $p \wedge (q \vee r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. (CO1,L 2)

(or)

(b) Show that if n is a positive integer, then 1+2+----------+n= n (n+1)/2 (CO1,L2)

2. (a) Solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$ (CO2,L3)

(or)

(b) Find the number of solutions of e1+e2+e3=17 where e1,e2 and e3 are non negative integers with $2 \le e1 \le 5$, $3 \le e2 \le 6$, and $4 \le e3 \le 7$. (CO2,L3)

3. (a) Define *Symmetric* and *Anti Symmetric* relations and give an example of each. (CO3,L2)

(or)

(b) Write *Dijkstra's Algorithm*(CO3,L2)

4. (a) State *Algorithms of Probability*? (CO4,L2)

(or)

(b) Explain *(i) Discrete Random Variables (ii) Continuous Random Variables.*(CO4,L2)

5. (a) Explain Procedure for Testing of Hypothesis.(CO5,L2)

(or)

(b) Explain *(i) Type-I Error (ii) Type-II Error.* (CO5,L2)

SECTION–B

Answer all questions: **5 × 10 = 50 Marks**

6. (a) (i) prove that $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology using truth table. (CO1,L2)

(ii) Prove that $\neg \forall x(p(x) \to Q(x))$ and $\exists x(p(x) \wedge \neg Q(x))$ are logically equivalent. (CO1,L2)

(b) Use mathematical induction to prove that $n^3 - n$ is divisible by 3, where n is a positive integer. (CO1,L2)

7. (a) Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 6$? (CO2,L2)

(or)

(b) Use generating functions to find the number of r combinations from a set with n elements when repetition of elements is allowed. (CO2,L2)

8. (a) Show that the relation R on a set A is transitive if and only if $R^n \subseteq R$ for n=1,2,3,…. (CO3,L2)

(or)

(b) Show that a connected multigraph with atleast two vertices has an Euler circuit if and only if its vertices have even degree. (CO3,L2)

9. (a) If 5% of the electric bulbs manufactured by a company are defective use Poisson distribution to find the probability that in a sample of 100 bulbs (i) none is defective,(ii) 5 bulbs will be defective (Given ; $e^{-5} = 0.007$) (CO4,L3)

(or)

(b) In a bolt factory machines A, B, C manufacture respectively 25%, 35% and 40% of the total of their output 5, 4, 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C? (CO4,L3)

10.(a) A machine puts out 16 imperfect articles in a sample of 500. After machine is overhauled, it puts out 3 imperfect articles in a batch of 100. Has the machine improved? (CO5,L3)

(or)

(b) Given the following information relating to two places, A and B, test whether there is any significant difference between their mean wages. (CO5,L3)

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	A	B
Mean Wages (Rs)	47	49
Standard Deviation (Rs.)	28	40
Number of Workers	1000	1500