P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010
Reaccredited at 'A+' level by NAAC
Autonomous & ISO 9001:2015 Certified

Title of the Course: REAL ANALYSIS - I
Semester : I

Course Code	22MA1T1	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2020-2021	Year of offering : 2022-23	Year of Revision: ---	Percentage of Revision :---

Course Objective: The main objective of this course is to develop problem solving skills and knowledge on the basic concepts of continuity, differentiation, Riemann-Stieltjes integrals, Improper integrals.

Course Outcomes: After successful completion of this course, students will be able to

CO-NO	COURSE OUTCOME	BTL	PO	PSO
CO1	understand the properties of continuous functions.	**K3**	**7**	**1**
CO2	understand the properties of differentiable functions.	**K3**	**7**	**1**
CO3	test the Riemann- Stieltjes integrability of bounded functions and their properties.	**K3**	**5**	**2**
CO4	understand the effect of uniform convergence on the limit function with respect to continuity, differentiability and integrability.	**K3**	**4**	**1**
CO5	test the convergence of improper integrals.	**K3**	**1**	**2**

Mapping of Course Outcomes:

CO-PO-PSO MATRIX										
	CO-PO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
22MA1T1	**CO1**							**2**	**2**	
	CO2							**2**	**2**	
	CO3					**3**				**3**
	CO4				**2**				**2**	
	CO5	**3**								**3**

UNIT-I

Continuity: Limits of functions- continuous functions- Continuity and Compactness- Continuity and Connectedness- Discontinuities.
[4.1 to 4.27 of chapter 4 of Text Book1]

UNIT-II

Differentiation:
Derivative of a Real Function- Mean value theorems- The Continuity of Derivatives- L' Hospital's rule- Derivatives of higher order- Taylor's theorem.
[5.1 to 5.15 of chapter 5 of Text Book1]

UNIT-III

The Riemann - Stieltjes Integral: Definition and Existence of Integral-Properties of the integral -Integration and Differentiation –Integration of vector-valued functions - Rectifiable Curves. [Chapter-6 of Text Book-1]

UNIT-IV

Sequences and Series of functions: Discussion of main problem - Uniform convergence – Uniform convergence and continuity – Uniform Convergence and Integration – Uniform Convergence and Differentiation – Equicontinuous families of functions – The Stone - Weierstrass Theorem.[7.1 to 7.26 of Text Book 1]

UNIT-V

Improper Integrals: Introduction – Integration of unbounded Functions with Finite limits of Integration – Comparison Tests for Convergence at a of $\int_a^b f dx$ - Infinite range of Integration – Integrand as a Product of Functions. [Chapter-11 of Text Book-2]

PRESCRIBED BOOKS:

1. Walter Rudin, "**Principles of Mathematical Analysis**", Student Edition 1976, McGraw-Hill International Publishers.
2. S.C. Malik and Savita Aurora, " **Mathematical Analysis",** Fourth edition, New Age International Publishers.

REFERENCE BOOK:

1.Tom. M. Apostol, "**Mathematical Analysis**" second Edition, Addison Wesley Publishing Company.

Course has Focus on : Foundation

Websites of Interest: 1. www. nptel.ac.in
2. www.epgp.inflibnet.ac.in
3. www.ocw.mit.edu

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA

(An Autonomous college in the jurisdiction of Krishna University)

M. Sc. Mathematics

First Semester

REAL ANALYSIS – I -22MA1T1

Time: 3 Hours **Max. Marks : 70**

SECTION- A

Answer all questions. **(5 X 4=20)**

1 (a) Let $f(x) = (1/x), x \neq 0$

$= 0, x = 0.$

Examine the continuity of the function f(x) on R. (CO1, K2)

(OR)

(b) If f is a continuous mapping of a compact metric space X into a metric space Y, then prove that f(X) is closed and bounded. (CO1, K2)

2 (a) Prove that every differentiable function on (a, b) is continuous on (a, b). (CO2, K2)

(OR)

(b) State and prove mean value theorem. (CO2, K2)

3 (a) Show that $\underline{\int_a^b} f d\alpha \leq \overline{\int_a^b} f d\alpha$. (CO3, K2)

(OR)

(b) State and prove fundamental theorem of calculus. (CO3, K2)

4 (a) Differentiate Pointwise convergence and Uniform convergence of sequence of functions. (CO4, K3)

(OR)

(b) For every interval [a, -a], prove that there is a sequence of real polynomials P_n such that $P_n(0) = 0$ and $\lim_{n\to\infty} P_n(x) = |x|$ uniformly on [a, -a] (CO4, K3)

5 (a) Examine the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x}}$ (CO5, K4)

(OR)

(b) Examine the convergence of $\int_0^\infty \sin x dx$ (CO5, K4)

SECTION- B

Answer all questions. All questions carry equal marks. **(5X10=50)**

6 (a) Show that a mapping f of a metric space X into a metric space Y is continuous if and only if $f^{-1}(V)$ is open in X, for every open set V in Y. (CO1, K3)

(OR)

(b) If f is a continuous mapping of a compact metric space X into a metric space Y, and E is a connected subset of X, then prove that f(E) is connected. (CO1, K3)

7 (a) Suppose f is continuous on [a, b], $f^1(x)$ exists at $x \in [a, b]$, g is defined on an interval I which contains the range of f, and g is continuous at f(x). If h(t) =g(f(t)), then prove that h is differentiable at x and $h^1(t) = f^1(g(x))g^1(x)$. (CO2, K2)

(OR)

(b) State and Prove Taylor's theorem. (CO2, K2)

8 (a) If f is monotonic on [a, b] and if α is continuous on [a, b] then show that $f \in R(\alpha)$.

(CO3, K3)

(OR)

(b) If γ^{1} is continuous on [a, b] then show that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma^1(t)| dt$.

(CO3, K3)

9 (a) If $\{f_n\}$ is sequence of continuous functions on E and if $f_n \rightarrow$ f uniformly on E, then show that f is continuous on E. (CO4, K4)

(OR)

(b) State and prove Stone – Weierstrass theorem. (CO4, K4)

10 (a) Test the convergence of the integral $\int_0^1 \frac{dx}{(x-a)^n}$ for n<1. (CO5, K4)

(OR)

(b) Show that if f and g are positive and $f(x) \leq g(x)$, for all x in [a, X] and $\int_a^\infty g(x)dx$ converges, then $\int_a^\infty f(x)dx$ converges. (CO5, K4)

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