

UNIT-I

Linear Equation of the first order: Introduction, Linear equations of the first order, The equation $y' + ay = 0$, The equation $y' + ay = b(x)$, The general linear equations of the first order, Linear Equations with constant coefficients, The homogeneous equation of order n , Initial value problems for n th order equations.

[Chapter 1 of Text Book(1) and Sections 7, 8 of Chapter 2 of Text book(1)]

UNIT-II

Linear Equations with Constant Coefficients: The non - homogeneous equation of order n , A special method for solving the non homogeneous equation.

Linear equations with variable coefficients: Introduction, Initial value problems for the homogeneous equations, Solution of the homogeneous equations, The Wronskian and linear independence.

[Sections 10,11 of Chapter 2 and Sections 1,2,3,4 of Chapter 3 of Text book(1)]

UNIT-III:

Solutions in Power series: Introduction– Second order Linear Equations with Ordinary points – Legendre equation and Legendre Polynomials – Second order equations with regular singular points – Properties of Bessel functions.

[Sections 3.1 to 3.5 of Chapter 3 of Text Book(2)]

UNIT-IV:

Systems of Linear Differential Equations: Introduction - Systems of first order equations - Model of arms competitions between two nations - Existence and uniqueness theorem - Fundamental Matrix - Non homogeneous linear systems - Linear systems with constant coefficients.[Sections 4.1 to 4.7 of Chapter 4 of Text Book (2)]

UNIT-V:

Existence and Uniqueness of solutions: Introduction – Successive approximations – Picard's theorem. [Sections 5.1 to 5.4 of chapter 5 of Text Book(2)]

PRESCRIBED BOOKS :

1. Earl.A. Coddington “*An Introduction to Ordinary Differential Equations*” , PHI.
2. S.G. Deo, V. Lakshmi kantham and V. Raghavendra “*Text Book of Ordinary Differential Equations*, Second edition, Tata McGraw Hill Pub., New Delhi, 1997.

REFERENCE BOOKS :

1. G.F. Simmons, *Differential equations with Applications and Historical Notes*, Second Edition , Tata Mc Graw Hill, (2003).
2. D. Somasundaram, “*Theory of Ordinary Differential Equations*”, Narosa Publications, 2001.

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA

(An autonomous college in the jurisdiction of Krishna University)

M. Sc. Mathematics

First Semester

ORDINARY DIFFERENTIAL EQUATIONS – 22MA1T2

Time: 3 hours

Max. Marks: 70

SECTION-A

Answer all questions.

(5x4=20)

1 a) Solve $y' - 2y = 1$

(CO1, L3)

(OR)

b) Solve $y' + e^x y = 3e^x$

(CO1, L3)

2 a) Write the characteristic polynomial of $y''' - 3y'' + 3y' - y = 0$ and find its roots.

(CO2, L2)

(OR)

b) Define homogeneous and non-homogeneous differential equations with examples.

(CO2, L2)

3 a) Express $f(t) = 1 + t + t^2$ in terms of Legendre series.

(CO3, L3)

(OR)

b) Show that $P_n(1) = 1$ and $P_n(-1) = (-1)^n$.

(CO3, L3)

4 a) Define fundamental matrix of the system of linear differential equations and give an example.

(CO4, L1)

(OR)

b) State Existence and uniqueness theorem for first order linear differential equation.

(CO4, L1)

5 a) State Lipschitz condition.

(CO5, L2)

(OR)

b) Compute first two successive approximations of the equation $x' = x$, $x(0) = 1$ (CO5, L2)

SECTION – B

Answer all questions. All questions carry equal marks.

(5X10=50)

6. a) Find the solution ϕ of $x^2 y' + 2xy = 1$ satisfying $\phi(2) = 2\phi(1)$

(CO1, L2)

(OR)

b) Consider the equation $y''' - 4y' = 0$. Compute three linearly independent solutions and Wronskian of the solutions. Also find the solution Φ satisfying $\Phi(0) = 0$, $\Phi'(0) = 1$, $\Phi''(0) = 0$.

(CO1, L2)

7.a) Compute the solution of the equation $y''' + y'' + y' + y = 1$, satisfying

$$\psi(0) = 0, \psi'(0) = 1 \text{ and } \psi''(0) = 0. \quad (\text{CO2, L3})$$

(OR)

b) Find two linearly independent solutions of the equation

$$y^{11} + \frac{1}{x} y' - \frac{1}{x^2} y = 0. \quad (\text{CO2, L3})$$

8 a) Show that the Legendre polynomials are given by

$$P_n(t) = \frac{1}{2^n n!} \frac{d^n}{dt^n} (t^2 - 1)^n \quad (\text{CO3, L2})$$

(OR)

b) Show that $\frac{d}{dt} [t^p J_p(t)] = t^p J_{p-1}(t)$

$$\text{and } \frac{d}{dt} [t^{-p} J_p(t)] = -t^{-p} J_{p+1}(t) \quad (\text{CO3, L2})$$

9 a) Find the fundamental matrix for $x' = Ax$ where $A = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$ (CO4, L3)

(OR)

b) Determine e^{At} for the system $x' = Ax$ where $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix}$ (CO4, L3)

10. a) State and prove Picard's theorem. (CO5, L2)

(OR)

b) Find the first three successive approximations of the equation $x' = e^x$, $x(0) = 0$. (CO5, L2)
