

UNIT-I

Analytic Functions: Continuity- Derivatives- Differentiation Formulas-Cauchy-Riemann Equations-Sufficient conditions for Differentiability-Polar Coordinates- Analytic Functions- Harmonic Functions. [Sec 18 to 26 of Chapter 2 of the Prescribed Text Book [1]]

UNIT-II

Integrals: Derivatives of functions $w(t)$ -Definite integrals of functions $w(t)$ - Contours- Contour Integrals- Cauchy-Goursat theorem- Proof of the theorem- Simply Connected Domains- Multiply Connected Domains- Cauchy Integral Formula- An extension of Integral Formula- Some Consequences of the extension- Liouville's Theorem and the Fundamental Theorem of Algebra.

[Sec 37 to Sec 41 and Sec 46 to Sec 53 of chapter 4 of the Prescribed Text Book [1]]

UNIT-III

Series: Convergence of Sequences- Convergence of Series-Taylor's series – Proof of Taylor's theorem- Examples- Laurent's series – Proof of Laurent's Series- Examples.

[Sec 55 to 62 of Chapter-5 of the Prescribed Text Book [1]]

UNIT-IV

Residues and Poles: Isolated singular points- Residues – Cauchy's residue theorem- Residue at Infinity- the three types of isolated singular points - Residues at poles, Zero's of analytic function- Zeros and Poles- Evaluation of improper integrals.

[Sec 68 to 76 of chapter 6 and sec 78, 79 of chapter 7 of the Prescribed Text Book [1]]

UNIT-V

Argument principle- Rouche's theorem- Linear Transformations: The transformation $w=1/z$ - Mappings by $1/z$ - Linear fractional transformations - The transformation $w=\sin z$.

[Sec 86 & 87 of chapter 7, sec 90 to 93, 96 of chapter 8 of the Prescribed Text Book [1]]

Prescribed Text Book:

1. "Complex Variables and Applications", James Ward Brown, Ruel V. Churchill, McGraw-Hill International Editions, Eighth Edition.

Reference Books:

1. “Complex analysis for Mathematics and Engineering”, John H. Mathews and Russell W. Howell, Narosa Publishing house.
2. “Complex Variables”, H. S. Kasana, Prentice Hall of India.

Course has Focus on :Foundation

- Websites of Interest:**
1. www.nptel.ac.in
 2. www.epgp.inflibnet.ac.in
 3. www.ocw.mit.edu

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA
(An autonomous college in the jurisdiction of Krishna University)
M. Sc. Mathematics
Second Semester

COMPLEX ANALYSIS– 22MA2T1

Time: 3 hours

Max. Marks: 70

SECTION-A

Answer all questions. All questions carry equal marks. (5x4=20)

1 a) Check the differentiability of $f(z) = \bar{z}$ (CO1, L2)

(OR)

b) Show that the function $f(z) = z^2$ is an entire function. (CO1, L2)

2 a) Evaluate $\int_c f(z)$, where $f(z) = 2e^{i\theta}$ and $0 \leq \theta \leq 2\pi$. (CO2, L2)

(OR)

b) Define simply connected domain and multiply connected domain.

Give an example for each. (CO2, L2)

3 a) Find the Taylor's series expansion of the function $f(z) = 1 / (1 + z)$ at the point 0. (CO3, L3)

(OR)

b) Find the Laurent series expansion of the function $f(z) = \frac{1}{z(1+z)}$ at the point 0 in the domain $1 < |z| < \infty$ (CO3, L3)

4 a) Find the residue of the function $f(z) = 2z / (z - 1)^2$ at $z = 1$. (CO4, L2)

(OR)

b) Find the residue of the function $f(z) = \frac{1}{z+z^2}$ at $z = 0$ (CO4, L2)

5 a) Find the linear transformation that maps $(2, i, -2)$ onto $(1, i, -1)$ (CO5, L2)

(OR)

b) Find the fixed points of $f(z) = \frac{z-1}{z+1}$ (CO5, L2)

SECTION – B

Answer all questions. All questions carry equal marks. (5X10=50)

6 a) Suppose that the complex function $f(z) = u + iv$ is differentiable at $z_0 = x_0 + iy_0$, then prove that the first order partial derivatives of 'u' and 'v' are exist and satisfies Cauchy Riemann equations $u_x = v_y$ and $u_y = -v_x$ at (x_0, y_0) . (CO1, L2)

(OR)

b) If a function $f(z) = u + iv$ is analytic in a domain D, then show that u and v are

harmonic in D. Also find a harmonic conjugate of $u(x, y) = y^3 - 3x^2 y$ (CO1, L2)

7 a) State and Prove Cauchy-Goursat Theorem. (CO2, L3)

(OR)

b) State and Prove Cauchy Integral formula. (CO2, L3)

8 a) State and prove Taylor's theorem. (CO3, L3)

(OR)

b) State and prove Laurent's theorem. (CO3, L3)

9 a) State and prove Cauchy's residues theorem. (CO4, L3)

(OR)

b) Using residue theorem, evaluate the improper integral $\int_0^{\infty} \frac{x^2}{x^6 + 1} dx$ (CO4, L3)

10 a) State and prove Rouché's Theorem. (CO5, L3)

(OR)

b) Discuss the transformation $w = 1/z$ (CO5, L3)
