

P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010 Reaccredited at 'A+' level by NAAC Autonomous&ISO 9001:2015 Certified

Title of the Course: COMPLEX ANALYSISSemester: II

Course Code	22MA2T1	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2020-21	Year of offering : 2022-23	Year of Revision: 	Percentage of Revision :

Course Objectives:

The main objective of the course is to learn the basic properties of complex numbers, analytical functions, differentiation and integration of complex valued functions.

CO-NO	COURSE OUTCOME	BTL	РО	PS O
CO1	use the Cauchy-Riemann equations to find the derivative of a complex valued function.	K3	1	2
CO2	Apply Cauchy integral formula to evaluate complex contour integrals.	K3	7	2
CO3	find power series representations of analytic functions.	К3	3	2
CO4	classify singularities and evaluate complex integrals using the residue theorem.	K3	7	2
CO5	Understand Rouche's theorem and Linear Transformations.	K3	1	1

Mapping of Course Outcomes:

CO-PO-PSO MATRIX										
	CO- PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PSO1	PSO2
	CO1	2								3
22MA2T1	CO2							3		3
	CO3			3						3
	CO4							3		3
	CO5	3							2	

UNIT-I

Analytic Functions: Continuity- Derivatives- Differentiation Formulas-Cauchy-Riemann Equations-Sufficient conditions for Differentiability-Polar Coordinates- Analytic Functions-Harmonic Functions. [Sec 18 to 26 of Chapter 2 of the Prescribed Text Book [1]]

UNIT-II

Integrals: Derivatives of functions w(t)-Definite integrals of functions w(t)- Contours-Contour Integrals- Cauchy-Goursat theorem- Proof of the theorem- Simply Connected Domains- Multiply Connected Domains- Cauchy Integral Formula- An extension of Integral Formula- Some Consequences of the extension- Liouville's Theorem and the Fundamental Theorem of Algebra.

[Sec 37 to Sec 41 and Sec 46 to Sec 53 of chapter 4 of the Prescribed Text Book [1]]

UNIT-III

Series: Convergence of Sequences- Convergence of Series-Taylor's series – Proof of Taylor's theorem- Examples- Laurent's series – Proof of Laurent's Series- Examples. [Sec 55 to 62 of Chapter-5 of the Prescribed Text Book [1]]

UNIT-IV

Residues and Poles: Isolated singular points- Residues – Cauchy's residue theorem- Residue at Infinity- the three types of isolated singular points - Residues at poles, Zero's of analytic function- Zeros and Poles- Evaluation of improper integrals.

[Sec 68 to 76 of chapter 6 and sec 78, 79 of chapter 7 of the Prescribed Text Book [1]]

UNIT-V

Argument principle- Rouche's theorem- Linear Transformations: The transformation w=1/z -Mappings by 1/z - Linear fractional transformations - The transformation $w=\sin z$. [Sec 86 &87 of chapter7, sec 90 to 93, 96 of chapter 8 of the Prescribed Text Book [1]]

Prescribed Text Book:

1. "Complex Variables and Applications", James Ward Brown, Ruel V. Churchill, McGraw-Hill International Editions, Eighth Edition.

Reference Books:

- "Complex analysis for Mathematics and Engineering", John H. Mathews and Russel W, Howell, Narosa Publishing house.
- 2. "Complex Variables", H. S. Kasana, Prentice Hall of India.

Course has Focus on :Foundation

Websites of Interest: 1. www. nptel.ac.in

- 2. www.epgp.inflibnet.ac.in
- 3. <u>www.ocw.mit.edu</u>

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA (An autonomous college in the jurisdiction of Krishna University) M. Sc. Mathematics Second Semester

COMPLEX ANALYSIS- 22MA2T1

Time: 3 hours

Max. Marks: 70

SECTION-A

Answer all questions. All questions carry equal marks.	(5x4=20)		
1 a) Check the differentiability of $f(z) = \overline{z}$	(CO1, L2)		
(OR)			

b) Show that the function $f(z) = z^2$ is an entire function. (CO1, L2)

2 a) Evaluate
$$\int_{c} f(z)$$
, where $f(z) = 2e^{i\theta}$ and $0 \le \theta \le 2\pi$. (CO2, L2)

(OR)

b) Define simply connected domain and multiply connected domain.

Give an example for each. (CO2, L2)

3 a) Find the Taylors series expansion of the function f(z) = 1 / (1 + z) at the point 0.

(CO3, L3)

(OR)

- b) Find the Laurent series expansion of the function $f(z) = \frac{1}{z(1+z)}$ at the point 0 in the domain $1 < |z| < \infty$ (CO3, L3)
- 4 a) Find the residue of the function $f(z) = 2z / (z-1)^2$ at z = 1. (CO4, L2)

(OR)

b) Find the residue of the function $f(z) = \frac{1}{z+z^2}$ at z = 0 (CO4, L2)

5 a) Find the linear transformation that maps (2, i, -2) onto (1, i, -1) (CO5, L2)

(OR)

b) Find the fixed points of
$$f(z) = \frac{z-1}{z+1}$$
 (CO5, L2)

SECTION – B

Answer all questions. All questions carry equal marks. (5X10=50)

6 a)Suppose that the complex function f (z) = u + iv is differentiable at z₀=x₀+iy₀, then prove that the first order partial derivatives of 'u' and 'v' are exist and satisfies Cauchy Riemann equations u_x = v_y and u_y = -v_x at (x₀,y₀). (CO1, L2)

(OR) b) If a function f(z) = u + i v is analytic in a domain D, then show that u and v are

harmonic in D. Also find a harmonic conjugate of $u(x, y) = y^3 - 3x^2 y$ (CO1, L2)

7 a) State and Prove Cauchy-Goursat Theorem.	(CO2, L3)		
(OR)			
b) State and Prove Cauchy Integral formula.	(CO2, L3)		

- 8 a) State and prove Taylor's theorem. (CO3, L3) (OR)
 - b) State and prove Laurent's theorem. (CO3, L3)
- 9 a) State and prove Cauchy's residues theorem. (CO4, L3) (OR)

b) Using residue theorem, evaluate the improper integral
$$\int_{0}^{\infty} \frac{x^2}{x^6 + 1} dx$$
 (CO4, L3)

10 a) State and prove Rouche's Theorem. (CO5, L3)

(OR)

b) Discuss the transformation w = 1/z (CO5, L3)