P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010
Reaccredited at 'A+' level by NAAC
Autonomous&ISO 9001:2015 Certified

Title of the Course: LATTICE THEORY
Semester : II

Course Code	22MA2T4	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2020-21	Year of offering : 2022-23	Year of Revision: 2022-23	Percentage of Revision : 5%

Course Objectives : The aim of this course is to understand the concepts ofPartly Ordered Sets, Complete lattices, Distributive lattices, Boolean algebras and classical propositional logic.

CO-NO	COURSE OUTCOME	BTL	PO	PSO
CO1	Understand partially ordered sets	**K3**	**3**	**2**
CO2	Analyze the relationship between posets and lattices	**K3**	**3**	**2**
CO3	Understand complete lattices and closure operations	**K3**	**7**	**2**
CO4	Characterize modular and distributive lattices	**K3**	**7**	**2**
CO5	Understand Boolean algebras and Boolean rings	**K3**	**1**	**1**

Mapping of Course Outcomes:

CO-PO-PSO MATRIX										
	CO-PO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
22MA2T3	**CO1**			**2**						**3**
	CO2			**3**						**3**
	CO3							**3**		**3**
	CO4							**3**		**3**
	CO5	**3**							**2**	

UNIT –I

Partly Ordered Sets: Set Theoretical Notations, Relations, Partly Ordered Sets, Diagrams, Special Subsets of a Partly Ordered Set, Length, Lower and Upper Bounds, The Minimum and Maximum Condition, The Jordan–Dedekind Chain Condition, Dimension Functions.
[Sections 1 to 9 of chapter I of Prescribed Book [1]]

UNIT – II

Lattices in General: Algebras, Lattices, The Lattice Theoretical Duality Principle, Semilattices, Lattices as Partly Ordered Sets, Diagrams of Lattices, Sublattices, Ideals, Bound Elements of a Lattice, Atoms and Dual Atoms, Complements, Relative Complements, Semicomplements, Irreducible and Prime Elements of a Lattice, The Homomorphism of a Lattice, Axiom Systems of Lattices. [Sections 10 to 21 of chapter II of Prescribed Book [1]]

UNIT – III

Complete Lattices: Complete Lattices, Complete Sublattices of a Complete Lattice, Conditionally Complete Lattices, σ-Lattices, Compact Elements, Compactly Generated Lattices, Subalgebra Lattice of an Algebra, Closure Operations, Galois Connections, Dedekind Cuts, Partly Ordered Sets as Topological Spaces.
[Sections 22 to 29 of chapter III of Prescribed Book [1]]

UNIT – IV

Distributive and Modular Lattices: Distributive Lattices, Infinitely Distributive and Completely Distributive Lattices, Modular Lattices, Characterization of Modular and Distributive Lattices by their Sublattices, Distributive Sublattices of Modular Lattices, The Isomorphism Theorem of Modular Lattices, Covering Conditions, Meet Representations in Modular and Distributive Lattices. [Sections 30 to 36 of chapter IV of Prescribed Book [1]]

UNIT-V

Boolean Algebras: Boolean Algebras, De Morgan Formulae, Complete Boolean Algebras, Boolean Algebras and Boolean Rings, The Algebra of Relations, The Lattice of Propositions, Valuations of Boolean Algebras. [Sections 42 to 47 of chapter VI of Prescribed Book [1]]

PRESCRIBED BOOK: Gabor Szasz, *Introduction to Lattice Theory*, Acadamic press, 1963.

REFERENCE BOOK: G. Birkhoff, *Lattice Theory***,** Third Edition**,** Colloquium publications**,** Vol. 25, American Mathematical Society,1995.

Course has Focus on : Foundation

Websites of Interest:
1. www. nptel.ac.in
2. www.epgp.inflibnet.ac.in
3. www.ocw.mit.edu

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA

(An autonomous college in the jurisdiction of Krishna University)

M. Sc. Mathematics

Second Semester

LATTICE THEORY-22MA2T4

Time: 3 hours **Max. Marks: 70**

SECTION-A

Answer all questions. **(5x4=20)**

1 (a) Define a Partly ordered set. Prove that the set of all real numbers is a partly ordered set with respect to natural ordering. (CO1, L1)

(OR)

(b) Define JDCC and give an example of a partly ordered set satisfying JDCC. (CO1, L1)

2 (a) Define (i) Meet irreducible element (ii) Join irreducible element and give examples of each. (CO2, L2)

(OR)

(b) Define a sublattice, ideal of a lattice. Prove that every sublattice is an ideal. (CO2, L2)

3 (a) Define closure operation. Prove that every maximal element is closed under a closure operation. (CO3, L2)

(OR)

(b) Define complete lattice. Prove that every complete lattice is bounded. (CO3, L2)

4 (a) Define a Distributive lattice and Modular lattice. Prove that every distributive lattice is modular. (CO4, L2)

(OR)

(b) Define transposed interval and covering conditions. Prove that every Modular Lattice satisfy covering conditions. (CO4, L2)

5 (a) Define a Boolean Ring. Prove that every Boolean ring is commutative. (CO5, L2)

(OR)

(b) State and prove De Morgan laws in a Boolean Algebra. (CO5, L2)

SECTION-B

Answer all questions. All questions carry equal marks. **(5X10=50)**

6 (a) If every subchain of a non-empty partly ordered set P has an upper bound, then prove that P contains a maximal element. (CO1, L2)

(OR)

(b) Prove that a partly ordered set can satisfy both the maximum and minimum conditions if and only if every one of its subchain is finite. (CO1, L2)

7 (a) Show that two lattices are isomorphic if and only if they are also order isomorphic. (CO2, L2)

(OR)

(b) (i) Show that every weakly complemented lattice is semicomplemented.

(ii) Show that every section complemented lattice bounded below is weakly complemented. (CO2, L2)

8 (a) If a lattice satisfies both the maximum and minimum conditions then show that it is complete. (CO3, L3)

(OR)

(b) Show that every element of a compactly generated lattice can be represented as a meet of finite number of meet irreducible elements. (CO3, L3)

9 (a) State and Prove Dedekind's Modularity criterion. (CO4, L4)

(OR)

(b) Show that all irredundant irreducible meet - representations of any element of a modular lattice have the same number of components. (CO4, L4)

10(a) For a Complete Boolean algebra B, show that the following conditions are equivalent.

(i) B is Completely meet- distributive.

(ii) B is Atomic.

(iii) B is isomorphic with the subset lattice of a set. (CO5, L3)

(OR)

(b) Show that the algebra of relations R (M) of a set M forms a complete Boolean algebra.

(CO5, L3)

****