P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010
Reaccredited at 'A+' level by NAAC
Autonomous & ISO 9001:2015 Certified

22MADSL202 : GALOIS THEORY
Semester : III

Course Code	22MA3D1	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2020-21	Year of offering : 2022-23	Year of Revision: 2022-23	Percentage of Revision : 5%

Course Objectives : To develop skills and knowledge on some of the basic concepts in Modules, Algebraic Extensions, Splitting fields, Polynomials solvable by radicals.

Course Outcomes: After successful completion of this course, students will be able to

CO-NO	COURSE OUTCOME	BTL	PO	PSO
CO1	Understand the concepts of modules, submodules, quotient modules, homomorphisms and characterize completely reducible modules.	**K3**	**1**	**1**
CO2	Derive and apply Gauss Lemma, Eisenstein criterion for irreducibility of Polynomials.	**K3**	**7**	**2**
CO3	understand the concept of normal and separable extensions.	**K3**	**1**	**1**
CO4	derive Fundamental theorem of Galois theory and related results.	**K3**	**1**	**2**
CO5	apply Galois theory to classical problems.	**K3**	**7**	**2**

Mapping of Course Outcomes:

CO-PO-PSO MATRIX										
	CO-PO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
22MA3D2	**CO1**	**3**							**2**	
	CO2							**3**		**3**
	CO3	**2**							**2**	
	CO4	**2**								**3**
	CO5							**3**		**3**

UNIT I

Modules: Definition and examples, sub modules and direct sums, R-homomorphisms and quotient modules, Completely reducible modules. (Sections 1 to 4 of chapter 14 of [1])

UNIT II

Algebraic Extensions of fields: Irreducible polynomials and Eisenstein's criterion, Adjunction of roots, Algebraic extensions, Algebraically closed fields.
(Sections 1 to 4 of chapter 15 of [1])

UNIT III

Normal and Separable extensions: Splitting fields, Normal extensions, Multiple roots, Finite fields, Separable extensions.
(Sections 1 to 5 of chapter 16 of [1])

UNIT IV

Galois Theory: Automorphism groups and fixed fields, Fundamental theorem of Galois theory, Fundamental theorem of algebra.
(Sections 1 to 3 of chapter 17 of [1])

UNIT V

Applications of Galois Theory to Classical Problems: Roots of unity and cyclotomic polynomials-Cyclic extensions- Ruler and compass constructions.
(Sections 1, 2, 5 of chapter 18 of [1])

PRESCRIBED TEXT BOOK:

1. Bhattacharya, P. B. Jain S. K and Nagpaul S. R, **Basic abstract algebra,** Second edition, Cambridge Press.

REFERENCE BOOKS:

1. Joseph Rotman, **Galois theory,** Second edition, Springer, 1998.

2. Artin M, **Algebra,** PHI, 1991.

3. David S Dummit and Richard M Foote, **Abstract Algebra**, Third Edition, Wiley Publications.

Course has Focus on : Foundation

Websites of Interest: 1. www. nptel.ac.in

2. www.epgp.inflibnet.ac.in

3. www.ocw.mit.edu

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M. Sc. Mathematics

Third Semester

22MA3D1 - GALOIS THEORY

Time: 3 hours **Max. Marks: 70**

SECTION A

Answer all questions. **(5x4=20M)**

1 a) Define an R-module, R-sub-module and give an example to each. (CO1, K2)

(or)

b) Let M be an R-module and RM= $\{\sum r_i m_i / r_i \in R, m_i \in M\}$ then show that RM is a sub module of M. (CO1, K2)

2. a) show that x^2–2 is a irreducible over Q. (CO2, K3)

(or)

b) Define irreducible polynomial and show that x^3-x-1 $\in$ Q[x] is irreducible over Q.

(CO2, K3)

3. a) Define splitting field of a polynomial f(x) over the field F and show that the degree of the extension of the splitting field of x^3-2 $\in$ Q[x] is 6. (CO3, K2)

(or)

b) Let f(x) be an irreducible polynomial over F then prove that f(x) has a multiple root

$f^1(x) = 0$. (CO3, K2)

4. a) If E is a finite extension of a field F, then prove that |G(E\F| $\leq$ [E:F]. (CO4, K2)

(or)

b) The group G(Q(α)/Q) where $\alpha^5 = 1$ and $\alpha \neq 1$ is isomorphic to the cyclic group of order 4. (CO4, K2)

5. a) Define primitive nth root of unity and n^{th} cyclotomic polynomial and given an example To each. (CO5, K2)

(or)

b) Show that the galois group of x^4+x^2+1 is the same as that of x^6-1 and is of order 2. (CO5, K2)

(P.T.O.)

SECTION B

Answer all questions. All questions carry equal marks. (5X10=50M)

6 a) Let f be an R-homomorphism of an R-module M into an R-module N. Then prove that M / kerf ≅ f(M) (CO1, K2)

(OR)

b) Let R be a ring with unity. Then prove that an R-module M is cyclic iff M ≅ R/ I, for some left ideal I or R. (CO1,K2)

7 a) State and prove Gauss lemma. (CO2,K3)

(OR)

b) Define algebraic element and algebraic extension of a field. If E is a finite extension of a field F, then prove that E is an algebraic extension of F. (CO2,K3)

8 a) Let E be a finite extension of a field F. Then prove that the following conditions are equivalent.

(i) $E=F(\alpha)$ for some $\alpha \varepsilon E$.

(ii) There are only a finite number of intermediate fields between F and E. (CO3, K2)

(OR)

b) Let $f(x) \epsilon F[x]$ be a polynomial of degree ≥ 1 with α as a root. Then prove that α is a multiple root if and only if $f'(\alpha) = 0$. (CO3,K3)

9 a) State and prove Dedikind lemma. (CO4,K4)

(OR)

b) State and prove the fundamental theorem of Galois theory. (CO4,K4)

10 a) Let F be a field contains a primitive nth root ω of unity, then prove the following are equivalent.

i) E is a finite cyclic extension of degree n over F.

ii) E is the splitting field of an irreducible polynomial $x^n - b \epsilon F[x]$. (CO5,K4)

(OR)

b) If a and b are constructible numbers, then prove that

i) ab is constructible.

ii) a / b, $b \neq 0$ is constructible. (CO5,K4)

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