P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010
Reaccredited at 'A+' level by NAAC
Autonomous & ISO 9001:2015 Certified

Title of the Course: LINEAR ALGEBRA
Semester : III

Course Code	22MA3D3	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2022-23	Year of offering : 2023-24	Year of Revision: 2023-24	Percentage of Revision : 5%

Course Objectives : The objective of this course is to provide students with an understanding of Mathematical concept on Linear Algebra that includes basic as well as advanced level with computational perspective. Linear System of Equations, Vector Spaces, Linear Transformations, Canonical Forms and Jordan Forms, Inner Product Spaces, Bilinear forms, the major components.

Course Outcomes: After successful completion of this course, students will be able to

CO-NO	COURSE OUTCOME	BTL	PO	PSO
CO1	solve linear equations.	**K3**	**3**	**2**
CO2	determine the basis and dimensions of Vector Spaces.	**K3**	**1**	**2**
CO3	understand the concept of Linear transformations.	**K3**	**3**	**1**
CO4	find the Eigen values and Eigen vectors of matrices.	**K3**	**5**	**2**
CO5	understand the concept of Inner product spaces.	**K3**	**3**	**1**

Mapping of Course Outcomes:

CO-PO-PSO MATRIX										
	CO-PO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
22MA3D3	**CO1**			**3**						**3**
	CO2	**3**								**3**
	CO3			**2**					**2**	
	CO4					**3**				**3**
	CO5			**2**					**2**	

UNIT – I

Linear Equations: Systems of Linear Equations, Matrices and Elementary row operations, Row Reduced Echelon Matrices, Matrix Multiplication, Invertible Matrices.

(Sections 1.2 to 1.6 of chapter 1 of prescribed book [1])

UNIT –II

Vector Spaces: Vector Spaces, Subspaces, Bases and Dimension, Coordinates.

(Sections 2.1 to 2.4 of chapter 2 of prescribed book [1])

UNIT – III

Linear Transformations: Linear Transformations, The algebra of Linear Transformations, Isomorphism, Representation of Transformations by Matrices, Linear Functionals.

(Sections 3.1 to 3.5 of chapter 3 of prescribed book [1])

UNIT - IV

Elementary Canonical Forms: Introduction, Characteristic Values, Annihilating Polynomials, Invariant Subspaces, Simultaneous Triangulation; Simultaneous Diagonalization, The Jordan Form.
(Sections 6.1 to 6.5 of chapter 6 and section 7.3 of chapter 7 of prescribed book [1])

UNIT – V

Inner Product Spaces: Inner Products, Inner Product Spaces, Unitary Operators, Normal Operators, Bilinear forms. (Sections 8.1, 8.2, 8.4, 8.5 of chapter 8 and section 10.1 of chapter 10 of prescribed book [1])

PRESCRIBED BOOK:

1.Kenneth Hoffman and Ray Kunze, Linear Algebra, Second edition, PHI publications (1992).

REFERENCE BOOKS:

1.Sheldon Axier, Linear Algebra Done Right, Springer Nature, 2015, third edition.

2.P.G.Bhattacharya, S.K.Jain, S.R.Nagpaul, , First Course in Linear Algebra, Wiley Eastern ltd., 1991.

3.K. B. Datta, Matrix and Linear Algebra, PHI Publications, 2006.

Course has Focus on : Skill Development

Websites of Interest: 1. https://onlinecourses.nptel.ac.in/noc23_ma17/preview

2. www.epgp.inflibnet.ac.in
3. www.ocw.mit.edu

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA

(An Autonomous College in the Jurisdiction of Krishna University)

M.Sc. Mathematics

Third Semester

LINEAR ALGEBRA – 22MA3D3

Time:3 hours **Max. Marks: 70**

SECTION A

Answer all questions. **(5x4=20)**

1 a) Define row-reduced echelon matrix and elementary matrix. Find all solutions of

$$A=\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{bmatrix}$$ (CO1, K2)

(OR)

b) Find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (CO1, K2)

2 a) Prove that the vectors (1, 0, -1), (1, 2, 1) and (0, -3, 2) are linearly independent in R^3.

(CO2, K3)

(OR)

b) If W is a proper subspace of a finite dimensional vector space V, then prove that W is finite dimensional and dim W< dim V. (CO2, K2)

3 a) Verify whether T: $R^2 \rightarrow R^2$ defined by $T(x_1, x_2) = (x_2, x_1)$ is a linear transformation. (CO3, K3)

(OR)

b) Let T be the linear operator on F^2 defined by $T(x_1, x_2) =(x_1+x_2, x_1)$. Find T^{-1}.

(CO3, K3)

4 a) Prove that similar matrices have same characteristic polynomial. (CO4, K4)

(OR)

b) Find the minimal polynomial of a linear operator represented by the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$ (CO4, K4)

5 a) Define an inner product space and give an example. (CO5, K2)

(OR)

b) If V and W are finite dimensional inner product spaces over the same field, then prove that V and W are isomorphic if and only if they have the same dimension.

(CO5, K2)

SECTION B

Answer the following questions. All questions carry equal marks. (5X10=50)

6 a) Find the solutions of the system of equations

$x_1 - x_2 + 2x_3 = 1$,

$2x_1 + 2x_3 = 1$,

$x_1 - 3x_2 + 4x_3 = 2$ and describe explicitly all solutions. (CO1, K2)

(OR)

b) For an nxn matrix A prove that the following are equivalent. (CO1, K2)

(i) A is invertible.

(ii) The homogenous system AX = 0 has only the trivial solution X =0

(iii) The system of equations AX=Y has a solution X for each nx1 matrix Y.

7 a) Prove that the subspace spanned by a non-empty subset S of a vector space V is the set of all linear combinations of vectors in S. (CO2, K3)

(OR)

b) If W_1 and W_2 are finite dimensional subspaces of a vector space V, then prove that

$\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$ (CO2, K3)

8 a) Let V and W be finite dimensional vectors spaces over a field F such that dim V= dim W.

If T is a linear transformation from V into W, prove that the following are equivalent.

(i) T is invertible.

(ii) T is non-singular.

(iii) T is onto, that is, the range of T is W. (CO3, K3)

(OR)

b) Let V be a finite dimensional vectors space over a field F and let W be a subspace of V. Then prove that dim W+ dim W^0 = dim V. (CO3, K3)

9 a) State and prove the Cayley-Hamilton theorem. (CO4, K3)

(OR)

b) Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is diagonalizable if and only if the minimal polynomial of T has the form $p = (x-c_1)\ldots(x-c_k)$, where $c_1,\ldots c_k$ are distinct elements of F. (CO4, K3)

10 a) If V is an inner product space, then prove that

(i) $\|c\alpha\| = |c|\|\alpha\|$ (ii) $\|\alpha\| > 0$, for $\alpha \neq 0$

(iii) $\|(\alpha \mid \beta)\| \leq \|\alpha\|\|\beta\|$ and (iv) $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$ (CO5, K4)

(OR)

b) Let U be a linear operator on an inner product space V. Then prove that U is unitary if and only if the adjoint operator U^* of U exists and UU*=U*U=1. (CO5, K4)

***