P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010
Reaccredited at 'A+' level by NAAC
Autonomous & ISO 9001:2015 Certified

Title of the Course: REAL ANALYSIS-II
Semester : III

Course Code	22MA3D4	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2020-21	Year of offering : 2023-24	Year of Revision: 2023-24	Percentage of Revision : 5%

Course Objectives: The objective of this course is to develop problem solving skills in the students and to have knowledge on Power Series, Functions of Several Variables, Inverse function theorem, Implicit function theorem and Differential forms.

Course Outcomes: After successful completion of this course, students will be able to

CO-NO	COURSE OUTCOME	BTL	PO	PSO
CO1	understand the properties of power series, Fourier series, Exponential, Trigonometric and Logarithmic functions.	**K3**	**1**	**1**
CO2	compute derivatives and integrals of real valued and vector valued functions of several variables.	**K3**	**3**	**2**
CO3	apply Inverse function theorem, Implicit function theorem.	**K3**	**5**	**2**
CO4	understand the concept of differential forms, the product and derivative.	**K3**	**3**	**1**
CO5	study the concept of simplexes.	**K3**	**3**	**1**

Mapping of Course Outcomes:

CO-PO-PSO MATRIX										
	CO-PO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
22MA3D4	**CO1**	**2**							**2**	
	CO2			**3**						**3**
	CO3					**3**				**3**
	CO4			**2**					**2**	
	CO5			**2**					**2**	

UNIT-I

Some Special Functions:

Power Series, The Exponential and Logarithmic functions, The Trigonometric functions, The Algebraic Completeness of the Complex field, Fourier Series.

[Sections 8.1 to 8.15 of Chapter 8 of Prescribed Book]

UNIT-II

Functions of Several Variables: Linear Transformations, Differentiation, Contraction Principle.

[Sections 9.1 to 9.23 of Chapter 9 of Prescribed Book]

UNIT-III

Functions of Several Variables (Continued):

Inverse function theorem, Implicit function theorem, The Rank theorem, Determinants, Derivatives of Higher order, Differentiation of Integrals.

[Sections 9.24 to 9.43 of Chapter 9 of Prescribed Book]

UNIT-IV

Integration of Differential Forms: Integration, Primitive Mappings, Partitions of Unity, Change of Variables , Differential Forms.

[Sections 10.1 to 10.25 of Chapter10 of Prescribed Book]

UNIT- V

Integration of Differential Forms (continued): Simplexes and chains, Stoke's theorem, Closed forms and Exact forms.

[Sections 10.26 to 10.41 of Chapter 10 of Prescribed Book]

PRESCRIBED BOOK: Walter Rudin, *Principles of Mathematical Analysis*, Third Edition, Tata Mc Graw Hill(1964).

REFERENCE BOOKS:

1. Tom.M. Apostol, *Mathematical Analysis*, Second Edition, Narosa Publishing House (2002).
2. D. Somasundaram, B. Choudary, *A First Course in Mathematical Analysis*, Narosa Publishing House(1996).

Course has Focus on : Foundation

Websites of Interest : 1. www. nptel.ac.in

2. www.epgp.inflibnet.ac.in

3. www.ocw.mit.edu

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA

(An autonomous college in the jurisdiction of Krishna University)

M. Sc. Mathematics

Third Semester

REAL ANALYSIS-II - 22MA3D4

Time: 3 hours **Max. Marks: 70**

SECTION - A

Answer all questions. **(5x4=20)**

1 a) Show that $(e^x)^1 = e^x$, for all real x. (CO1, K2)

(OR)

b) Prove that $E(z+2\pi i) = E(z)$, for any complex number z. (CO1, K2)

2 a) Define Linear Transformation. (CO2, K2)

(OR)

b) Show that dim $R^n = n$. (CO2, K2)

3 a) Show that if a Linear operator **A** on R^n is invertible then det $[A] \neq 0$. (CO3, K3)

(OR)

b) Let $f(0, 0) = 0$ and $f(x, y) = xy\,(x^2 - y^2)/\,(x^2 + y^2)$, then prove that $(D_{12} f)(0,0) = 1$ and $(D_{21} f)(0, 0) = -1$. (CO3, K3)

4 a) If γ is a 1-surface in R^3, with domain [0, 1] and $w = xdy+ydx$, then find $\int_\gamma \omega$ (CO4, K3)

(OR)

b) Let ω and λ be k and m- forms in an open set V respectively, then show that $(\omega + \lambda)_T = \omega_T + \lambda_T$. (CO4, K3)

5 a) Write a brief note on Affine-simplexes. (CO5, K2)

(OR)

b) Define exact form and closed form in R^n. Prove that every exact form is closed. (CO5, K2)

SECTION-B

Answer all questions. All questions carry equal marks. (5X10=50)

6 (a) Suppose $\sum_{n=o}^{\infty} c_n$ converges. Let $f(x) = \sum_{n=o}^{\infty} c_n x^n$ $(-1< x < 1)$. Then show that

$$\lim_{x\to 1} f(x) = \sum_{n=o}^{\infty} c_n .$$ (CO1, K2)

(OR)

(b) Show that

(i) The functions C and S are periodic, with period 2Π.

(ii) If 0<t<2Π, then $E(it) \neq 1$

(iii) If z is complex number with $|z| = 1$, there is a unique t in $[0,2\pi)$ such that E(it)=z. (CO1, K2)

7 (a) Show that a linear operator A on a finite dimensional vector space X is one –to - one if and only if the range of A is all of X . (CO2, K2)

(OR)

(b) Define a contraction mapping. State and prove contraction principle. (CO2, L2)

8 (a) State and prove Inverse function theorem. (CO3, K4)

(OR)

(b) Suppose f is defined on an open set $E \subset R^n$ and suppose that $D_1 f$, $D_{21} f$ and $D_2 f$ exists at every point of E, and $D_{21} f$ is continuous at some point $(a,b) \in E$. Then show that $D_{12} f$ exists and $(D_{12} f)(a,b) = D_{21} f(a,b)$. (CO4, K4)

9 (a) For every $f \in C(I^k)$, show that $L(f)= L^1(f)$ (CO4, K3)

(OR)

(b) Suppose E is an open set in R^n. If w is of class $\mathcal{C}''$ in E, then show that $d^2 w = 0$

(CO4, K3)

10 (a) State and prove Stoke's theorem. (CO5, K4)

(OR)

(b) Define an exact form. If $E \subset R^n$ is convex open set, $k \geq 1$, w is a k- form of class $\mathcal{C}'$ in E, and if $dw = 0$, then show that there is a (k-1)- form λ in E such that $w = d\lambda$.

(CO5, K4)

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