P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010
Reaccredited at 'A+' level by NAAC
Autonomous & ISO 9001:2015 Certified

Title of the Course: MEASURE AND INTEGRATION
Semester : III

Course Code	22MA3T1	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2020-21	Year of offering : 2023-24	Year of Revision: 2023-24	Percentage of Revision :5%

Course Objective : The objective of this course is to acquire knowledge on basic concepts of Outer measure, Measurable sets, Lebesgue Measure, Lebesgue Integral, Measurable Functions, and to extend these results and related concepts in a measure space.

Course Outcomes: After successful completion of this course, students will be able to

CO-NO	COURSE OUTCOME	BTL	PO	PSO
CO1	understand the concept of measure and properties of Lebesgue measure.	**K3**	**1**	**1**
CO2	study the properties of Lebesgue integral and compare it with Riemann integral.	**K3**	**7**	**1**
CO3	find the derivative of an Integral and Integral of a derivative for the functions of bounded variation.	**K3**	**3**	**2**
CO4	construct different measures for P(X) and study their properties.	**K3**	**7**	**2**
CO5	study the concept of integral with respect to product measure.	**K3**	**1**	**1**

Mapping of Course Outcomes:

CO-PO-PSO MATRIX										
	CO-PO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
22MA3T1	**CO1**	**2**							**2**	
	CO2							**3**	**2**	
	CO3			**3**						**3**
	CO4							**3**		**3**
	CO5	**2**							**2**	

UNIT-I

Lebesgue Measure: Introduction, Outer measure, Measurable sets and Lebesgue measure, A Non measurable set, Measurable functions, Littlewood's three principles. (Chapter3)

UNIT-II

The Lebesgue Integral: The Riemann Integral, The Lebesgue Integral of a bounded function over a set of finite measure, The Integral of a non-negative function, The general Lebesgue Integral. (Sections 4.1to 4.4 of Chapter4).

UNIT -III

Differentiation and Integration: Differentiation of monotone functions, Functions of bounded variation, Differentiation of an Integral, Absolute continuity. (Sections 5.1 to 5.4 of Chapter 5)

UNIT-IV

Measure and Integration: Measure spaces, Measurable functions, Integration, Signed Measures, The Radon-Nikodym theorem. (Sections 11.1 to11.6 of Chapter11)

UNIT-V

Measure and Outer Measure: Outer Measure and Measurability, The Extension theorem, Product measures.
(Sections 12.1, 12.2 &12.4 of Chapter12).

PRESCRIBED BOOK:

1. Royden H.L., **Real Analysis,** Third Edition, Pearson publishers.

REFERENCE BOOKS:

1. Halmos P.R, **Measure Theory,** Springer-Verlag, 1974.
2. Bogachev V.I, **Measure Theory** Springer-Verlag, 1997.

Course has Focus on : Foundation

Websites of Interest: 1. www. nptel.ac.in

2. www.epgp.inflibnet.ac.in

3. www.ocw.mit.edu

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA

(An Autonomous college in the jurisdiction of Krishna University)

M.Sc. Mathematics

Third Semester

MEASURE AND INTEGRATION -22MA3T1

Time:3 hours **Max. Marks: 70**

SECTION A

Answer all questions. **(5x4=20)**

1 a) Define Outer measure. If E is a subset of R such that $m^*(E) = 0$, prove that E is measurable. (CO1, K2)

(OR)

b) State and prove Littlewoods three principles. (CO1, K2)

2 a) If f and g are integrable over E, then prove that f+g is integrable and $\int_E f + g = \int_E f + \int_E g$ (CO2, K2)

(OR)

b) If f is integrable, then show that $|f|$ is integrable. (CO2, K2)

3 a) Show that a function f is of bounded variation iff f is the difference of two monotone real valued functions. (CO3, K2)

(OR)

b) Prove that every absolutely continuous function is of bounded variation on [a, b] (CO3, K2)

4 a) Define Positive set and Negative set with respect to a signed measure. Prove that every measurable subset of a positive set is positive. (CO4, K2)

(OR)

b) Prove that Hahn decomposition is not unique. (CO4, K2)

5 a) Prove that the union of two measurable sets is measurable. (CO5, K3)

(OR)

b) Prove that the set function μ^* is an outer measure. (CO5, K3)

SECTION B

Answer the following questions. All questions carry equal marks. (5X10=50)

6 a) State and Prove Egoroff's theorem. (CO1, K4)

(OR)

b) If$\{E_n\}$ is a decreasing sequence of measurable sets with mE_1 finite, then show that $m(\cup E_n) = \lim m(E_n)$. (CO1, K4)

7 a) State and Prove Lebesgue convergence theorem. (CO2, K3)

(OR)

b) Let f be a non-negative function which is integrable over a set E. Then Show that given $\epsilon > 0$, there is a $\delta > 0$ such that for every set $A \subset E$ with $mA < \delta$, $\int_A f < \epsilon$.

(CO2, K3)

8 a) State and Prove Vitali Covering Lemma. (CO3, K4)

(OR)

b) If f is absolutely continuous on [a, b] and $f^1(x)=0$ a.e., then show that f is constant.

(CO3, K4)

9 a) Define Positive set and Negative set with respect to a signed measure ν. Let E be a measurable set such that $0<\nu(E)<\infty$, then show that there is a positive set A contained in E with $\nu(A)>0$. (CO4, K3)

(OR)

b) State and prove the Jordan Decomposition Theorem. (CO4, K3)

10 a) State and prove the Caratheodary Extension Theorem. (CO5, K3)

(OR)

b) State and prove Fubini's Theorem. (CO5, L3)

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