PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
Siddhartha Nagar, Vijayawada–520010
Re-accredited at 'A+' by the NAAC

22MA4D2 : ANALYTICAL NUMBER THEORY
Semester : IV

Course Code	22MA4D2	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2021-22	Year of offering : 2023-24	Year of Revision: 2023-24	Percentage of Revision : 5%

Course Objectives : This course is introduced to develop problem solving skills and to acquire knowledge on concepts of Arithmetical functions, Dirichlet multiplication, Averages of Arithmetical functions and Congruences.

COURSE OUTCOME	Upon successful completion of this course, students will be able to:
CO1	understand the concepts of divisibility, congruence, dirichlet product and multiplicative functions.
CO2	understand the concepts of arithmetical functions and multiplicative functions.
CO3	understand some elementary theorems on the distribution of prime numbers.
CO4	understand the properties of congruences.
CO5	understand Chinese Remainder theorem and its applications.

Mapping of Course Outcomes:

	PO1	PO2	PO3	PO4	PO5	PO6	PO7
CO1	2	0	0	0	0	0	0
CO2	0	0	0	0	0	0	1
CO3	2	0	0	0	0	0	0
CO4	0	0	0	0	0	0	1
CO5	1	0	0	0	0	0	0

Unit	Learning Units	Lecture Hours
I	**Arithmetical Functions and Dirichlet Multiplication:** Introduction, The Mobius function μ(n), The Euler Totient function φ(n), A relation connecting φ and μ, A product formula for φ(n), The Dirichlet product of arithmetical functions, Dirichlet inverses and the Mobius inversion formula, The Mangoldt function Λ(n), Multiplicative functions, Multiplicative functions and Dirichlet multiplication, The inverse of a completely multiplicative function, Liouville's function λ(n), The divisor function $\sigma_\alpha(n)$, Generalized convolutions. (Sections 2.1to2.14of Chapter 2 of [1])	12
II	**Averages of Arithmetical Functions:** Introduction, The big oh notation. Asymptotic equality of functions, Euler's summation formula, Some elementary asymptotic formulas, The average order of d(n), The average order of divisor functions $\sigma_\alpha(n)$, The average order of φ(n), An application to the distribution of lattice points visible from the origin, The average order of μ(n) and of Λ(n), The partial sums of a Dirichlet product, Applications to μ(n) and Λ(n), Another identity for the partial sums of a Dirichlet product. (Sections 3.1to3.12of Chapter 3 of [1])	12
III	**Some Elementary Theorems on the Distribution of Prime Numbers:** Introduction, Chebyshev's functions ψ(x) and ϑ(x). Relations connecting ϑ(x) and π(x), Some equivalent forms of the prime number theorem, Inequalities of π(n) and $\boldsymbol{p_n}$, Shapiro's Tauberian theorem, Application of Shapiro's theorem, An asymptotic formula for the partial sums $\sum_{p\le x}(1/p)$, The Partial Sums of the Mobius function.(Sections 4.1to 4.9 of Chapter 4 of [1])	12
IV	**Congruences:** Definition and basic properties of congruences, Residue classes and complete residue systems, Linear congruences, Reduced residue systems and Euler- Fermat theorem, Polynomial congruences modulo p, Lagrange's theorem. (Sections 5.1 to 5.5 of Chapter 5 of [1])	12
V	Applications of Lagrange’s Theorem, Simultaneous linear congruences, The Chinese remainder theorem, Applications of the Chinese Remainder Theorem, Polynomial Congruences with prime power moduli. (Sections 5.6 to 5.9 of Chapter 5 of [1]).	12

PRESCRIBED TEXT BOOK:

1. Apostol Tom M, **Introduction to Analytic Number Theory,** New Delhi, Narosa Publishing House, 1998.

REFERENCE BOOK:

1. Hardy G.H. and Wright E.M, **An Introduction to the Theory of Numbers,** Oxford Press.

Course has focus on : **S**kill Development

Websites of Interest: 1. www. nptel.ac.in

2. www.epgp.inflibnet.ac.in

3. www.ocw.mit.edu

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M. Sc. Mathematics
Fourth Semester

22MA4D2 - ANALYTICAL NUMBER THEORY

Time: 3 hours **Max. Marks: 70**

SECTION A

Answer all questions. **(5x4=20)**

1 a) Define Mobius function $\mu(n)$, Euler -Totient function and Dirichlet product. (CO1,L1)

(OR)

b) Prove that every completely multiplicative function is multiplicative. (CO1,L1)

2 a) Define Euler's constant, Riemann zeta function and lattice points. (CO2,L1)

(OR)

b) Prove that the average order of $\sigma(n)$ is $\pi^2 n/12$ (CO2,L1)

3 a) Define Chebyshev's functions. (CO3,L2)

(OR)

b) Write equivalent forms of prime number theorem. (CO3,L2)

4 a) Show that the linear congruence $2x \equiv 3 \pmod 4$ has no solution. (CO4,L3)

(OR)

b) Prove that congruence is an equivalence relation. (CO4,L3)

5 a) State and prove Wilson's Theorem. (CO5,L3)

(OR)

b) Solve the congruence $5x \equiv 3 \pmod{24}$. (CO5,L3)

SECTION B

Answer all questions. All questions carry equal marks. **(5X10=50)**

6 a) State and prove Mobius Inversion formula. (CO1,L3)

(OR)

b) Show that if both g and $f * g$ are multiplicative, then f is also multiplicative. (CO1, L3)

7.a) State and prove Euler's summation formula. (CO2, L3)

(OR)

b) State and prove Legendre's Identity. (CO2,L3)

8 a) State and prove Abel's Identity. (CO3,L4)

(OR)

b) Show that the following relations are logically equivalent: (CO3,L4)

(i) $\lim_{x\to\infty} \frac{(\pi(x)\log x)}{x} = 1$ (ii) $\lim_{x\to\infty} \frac{\vartheta(x)}{x} = 1$ (iii) $\lim_{x\to\infty} \frac{\psi(x)}{x} = 1$

where $\pi(x)$ is the number of primes not exceeding x and $\vartheta(x)$ and $\psi(x)$ are Chebyshev's functions.

9 a) Assume (a, m) = d, where a, b and m are integers, m > 0. Then show that the linear congruence ax ≡ b (mod m) has solutions if and only if d/b. (CO4,L4)

(OR)

b) State and prove Euler-Fermat Theorem. (CO4,L4)

10 a) Show that for a prime p, all the coefficients of the polynomial $f(x) = (x-1)(x-2)\ldots(x-p+1)-x^{p-1}+1$ are divisible by p. (CO5,L3)

(OR)

b) State and prove the Chinese Remainder theorem. (CO5,L3)

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