PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
Siddhartha Nagar, Vijayawada–520010
Re-accredited at 'A+' by the NAAC

22MA4M1 : FUNCTIONAL ANALYSIS
Semester : IV

Course Code	22MA4M1	Course Delivery Method	Blended Mode
Credits	4	CIA Marks	30
No. of Lecture Hours / Week	4	Semester End Exam Marks	70
Total Number of Lecture Hours	60	Total Marks	100
Year of Introduction : 2021-22	Year of offering : 2023-24	Year of Revision: 2023-24	Percentage of Revision :5%

Course Objectives : The main objective of the course is to understand concepts of Normed spaces, Banach Spaces, Hilbert Spaces, operators on Hilbert spaces and applications of Hahn –Banach theorems, Open mapping theorem and Uniform boundedness theorem.

COURSE OUTCOME	**Upon successful completion of this course, students will be able to:**
CO1	understand the fundamental properties of metric spaces, normed and Banach Spaces.
CO2	understand the concepts of linear operators on normed spaces.
CO3	study the applications of Hahn-Banach theorem, open mapping theorem, uniform boundedness theorem.
CO4	understand concepts of Hilbert and Banach spaces and construct orthonormal sequences and series using Gram- Schmidt process.
CO5	understand Riesz representation theorem, self adjoint, unitary operators on Hilbert spaces and applications to bounded linear functional.

Mapping of Course Outcomes:

	PO1	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**
CO1	0	0	3	0	0	0	0
CO2	0	0	0	0	0	0	3
CO3	0	0	0	0	0	2	0
CO4	0	0	0	0	0	0	3
CO5	3	0	0	0	0	0	0

UNIT –I

Metric Spaces: Metric Space, Further Examples of Metric Spaces, Convergence, Cauchy Sequence, Completeness, Completion of Metric Spaces.

Normed Spaces, Banach Spaces: Vector Spaces, Normed Space, Banach Space, Further Properties of Normed Spaces, Finite Dimensional Normed Spaces and Subspaces, Compactness and Finite Dimension.

(Sections 1.1, 1.2, 1.4, 1.6 of Chapter 1 of [1] and Sections 2.1 to 2.5 of Chapter 2 of [1]]

UNIT –II

Linear Operators, Bounded and Continuous Linear Operators, Linear Functionals, Linear Operators and Functionals on Finite Dimensional Spaces, Strong and Weak Convergence, Convergence of sequences of Bounded Linear Operators and Functionals, Open Mapping Theorem, Closed Graph Theorem. (Sections 2.6 to 2.9 of Chapter 2 & 4.8, 4.9 and 4.12 of Chapter 4 of [1]).

UNIT –III

Hahn- Banach Theorem, Hahn – Banach Theorem for Complex Vector Spaces and Normed Spaces, Applications to Bounded Linear Functionals on C[a, b], Adjoint Operator, Reflexive Spaces, Category Theorem, Uniform Boundedness Theorem.

(Sections 4.2 to 4.7 Chapter 4 of[1]).

UNIT –IV

Inner product Spaces, Hilbert Spaces: Inner Product Space, Hilbert Space, Further Properties of Inner Product Spaces, Ortogonal Complements and Direct Sums, Orthonormal Sets and Sequences, Series related to Orthonormal Sets and Sequences.

(Sections 3.1 to 3.5 of Chapter 3 of [1])

UNIT V

Inner product Spaces, Hilbert Spaces: Total Orthonormal Sets and Sequences, Representation of Functionals on Hilber Spaces, Hilbert Adjoint Operator, Self Adjoint, Unitary and Normal Operators. (Sections 3.6 to 3.10 of chapter 3 of [1])

PRESCRIBED BOOKS:

[1] E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons (1989).

[2] A. E. Taylor, Functional Analysis, John Wiley and Sons (1958).

[3] P.K.Jain and O.P.Ahuja, Functional Analysis, New Age International Pvt. Ltd.(2011).

REFERENCE BOOK:

[1]S.K. Berberian, Introduction to Hilbert spaces, Oxford University Press, 1961.

[2] D. Somasundaram , A first course in functional Analysis, Narosa Publishing House, 2006.

Course has Focus on : Foundation (Elective Paper)

Websites of Interest:
1. www. nptel.ac.in
2. www.epgp.inflibnet.ac.in
3. www.ocw.mit.edu

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M. Sc. Mathematics
Fourth Semester
22MA4M1 – FUNCTIONAL ANALYSIS

Time: 3 hours **Max. Marks: 70**

SECTION - A

Answer all questions. All questions carry equal marks. **(5x4=20)**

1 (a) Prove that a subspace M of a complete metric space is complete if and only if M is closed in X. (CO1, L2)

(OR)

(b) Prove that a compact subset of a metric space is closed and bounded. (CO1, L2)

2 (a) Let T be a linear operator. Then prove that the null space N(T) and the range R(T) of T are vector spaces. (CO2, L3)

(OR)

(b) Prove that strong convergence implies weak convergence with the same limit in a normed space X. (CO2, L3)

3 (a) Prove that every finite dimensional normed space is reflexive. (CO3, L2)

(OR)

(b) Prove that the adjoint operator of a bounded linear operator is linear and bounded. (CO3, L2)

4 (a) State and prove Paralellogram equality in an inner product space. (CO4, L2)

(OR)

(b) Prove that every orthonormal set in an inner product space is linearly independent. (CO4, L2)

5 (a) If $\langle v_1, w\rangle = \langle v_2, w\rangle$, for all w in an innerproduct space, prove that $v_1 = v_2$. (CO5, L3)

(OR)

(b) Prove that the product of two self adjoint operators S and T on a Hilbert space H is self-adjoint if and only if ST = TS. (CO5, L3)

SECTION – B

Answer the following questions. All questions carry equal marks. (5X10=50)

6 a) State and prove Holder's inequality. (CO1, L3)

(OR)

b) Show that every finite dimensional subspace of a normed space is complete.(CO1, L3)

7 a) Prove that if a normed space X is finite dimensional, then every linear operator on X is bounded. (CO2, L3)

(OR)

b) State and prove the Open mapping theorem. (CO2, L3)

8 a) State and prove Hahn- Banach theorem for vector spaces. (CO3, L4)

(OR)

b) State and prove uniform boundedness theorem. (CO3, L3)

9 a) Let Y be any closed subspace of a Hilbert space H. Then prove that $H = Y \oplus Z$, where $Z = Y^{\perp}$ (CO4, L3)

(OR)

b) State and prove Bessel's inequality. (CO4, L3)

10 a) Prove that two Hilbert spaces are isomorphic if and only if they have the same Hilbert dimension. (CO5,L3)

(OR)

b) State and prove Riesz representation theorem. (CO5,L3)

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