PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
Siddhartha Nagar, Vijayawada–520010
Re-accredited at 'A+' by the NAAC

Course Code				**23STMAL236**			
Title of the Course				**Probability Foundations for Artificial Intelligence**			
Offered to: (Programme/s)				**B.Sc. Honours -Artificial Intelligence**			
L	**4**	**T**	**0**	**P**	**0**	**C**	**3**
Year of Introduction:		**2024-25**		**Semester: III**			**3**
Course Category:		**MAJOR**		**Course Relates to:**		**Local, Regional, National, Global**	
Year of Revision		**NA**		**Percentage:**		**NA**	
Type of the Course:				**SKILL DEVELOMENT**			
Crosscutting Issues of the Course:				**NA**			
Pre-requisites, if any				**23STMAL234**			

Course Description:

This course offers a thorough exploration of probability theory, random variables, and statistical distributions, with a focus on both theoretical foundations and practical applications. Students will begin by learning about random experiments, sample spaces, events, and the algebra of events, followed by the classical, statistical, and axiomatic definitions of probability. Topics such as conditional probability, independent events, and Bayes' theorem are introduced with real-world problems. The course covers discrete and continuous random variables, their probability mass, density, and cumulative distribution functions, along with properties and applications. Joint, marginal, and conditional distributions are examined for two-dimensional random variables, alongside transformations of random variables. Mathematical expectations, variances, covariances, and key theorems are discussed in depth, including the Cauchy-Schwartz inequality and Chebychev's inequality. Students will also explore moment generating functions (MGFs), probability generating functions (PGFs), and characteristic functions (CFs) with their properties. Finally, the course delves into the definitions, properties, and applications of key probability distributions, including Bernoulli, Binomial, Poisson, Geometric, and Normal distributions, providing a comprehensive statistical foundation for applied analysis.

Course Objectives:

S. No	COURSE OBJECTIVES
1	Understand and apply probability concepts
2	Analyze random variables and their distributions
3	Evaluate transformations and expectations
4	Work with generating functions
5	Explore probability distributions and their applicationsand their applications.

Course Outcomes

At the end of the course, the student will be able to…

NO	COURSE OUTCOME	BTL	PO	PSO
CO1	define and explain the fundamental concepts of probability and random experiments	**K2**	**2**	**1**
CO2	apply the addition and multiplication theorems of probability to solve real-world problems	**K3**	**2**	**1**
CO3	distinguish between discrete and continuous random variables and analyze their probability functions	**K4**	**2**	**1**
CO4	evaluate the properties of mathematical expectation, variance, and covariance using generating functions	**K5**	**2**	**1**
CO5	solve problems involving well-known discrete and continuous probability distributions	**K4**	**2**	**1**

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO-PSO MATRIX									
CO NO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
CO1		**2**						**2**	
CO2		**3**						**3**	
CO3		**3**						**3**	
CO4		**3**						**3**	
CO5		**3**						**3**	

Use the codes 3, 2, 1 for High, Moderate and Low correlation Between CO-PO-PSO respectively

Course Structure:

Unit– 1:Probability (12 hours)

Introduction, random experiments, sample space, events and algebra of events. Definitions of probability-classical, statistical and axiomatic. Conditional Probability, addition and multiplication theorem of probability- Problems , independent events, Bayes' theorem, and its applications.

Examples :

1. Predicting weather patterns based on historical data and statistical models.
2. Ensuring product reliability and consistency through statistical process control.

Exercise:

1. Coin Flipping**:** A classic for exploring probability. Students can predict outcomes, record results, and calculate experimental probability.
2. Dice Rolling: Like coin flipping, but with more outcomes. Students can explore different combinations and probabilities.

Unit–2: Random variables (12 hours)

Introduction, discrete and continuous, illustrations and properties of Random variables, probability mass function, probability density function and cumulative distribution functions and its properties. Two Dimensional random variables: Joint, marginal and conditional probability mass function and probability density function, independence of random variables. Transformation of one-dimensional random variable- Problems.

Examples:

1. Risk assessment: Estimating the probability of financial losses.
2. Portfolio management: Evaluating investment returns and risks.

Exercise:

1. Simulate rolling a die 100 times and count the frequency of each outcome.
2. Simulate bus arrival times for 10 buses and calculate average wait time.

Unit3: Mathematical Expectation & Generating Functions (12 hours)

Introduction- Mathematical expectation of a random variable-expected of function of a random variable. Properties of expectations. Addition and Multiplication theorems on expectation. Variance – Properties of variance and covariance- properties of covariance. Inequalities involving expectation- Cauchy – Schwartz. Definition of Moment Generating Function (m.g.f.)- properties- problems, Definition of Cumulate Generating Function (c.g.f.)-properties, Definition of Probability Generating Function (p.g.f.)- Properties, Definition of Characteristic Function (c.f.) – properties- and Chebychev's Inequality-Problems.

Examples:

1. Expected value in portfolio management.
2. Expected value in insurance pricing.

Exercise:

1. Students can calculate the expected value for each ticket based on its probability and prize.
2. They can then determine the expected total winnings based on the number of tickets purchased.

Unit–4: Discrete Probability Distributions (12 hours)

Discrete Uniform, Bernoulli, Binomial, Poisson, Geometric, Negative Binomial and Hyper-geometric distributions – Definitions, properties and applications.

Examples:

1. Bernoulli distribution: Modelling single trials with two outcomes (e.g., coin flips, success/failure).
2. Binomial distribution: Modelling the number of successes in a fixed number of independent trials (e.g., number of heads in 10 coin flips).

Exercise:

1. **Binomial Distribution Simulation:**

a. Simulate a binomial experiment (e.g., flipping a coin 10 times).
b. Calculate the probability of different numbers of successes.
c. Compare the simulated results with theoretical probabilities.

2. **Poisson distribution Application**:

a) Collect data on the number of customers arriving at a store per hour.
b) Fit a Poisson distribution to the data.
c) Use the Poisson distribution to estimate the probability of different numbers of customers.

3. **Geometric Distribution Problem:**

a) Solve word problems involving geometric distributions (e.g., finding the probability of getting the first head on the fourth coin toss).
b) Calculate the expected number of trials until the first success.

Unit 5: Continuous Probability Distributions (12 hours)

Rectangular, Exponential, Gamma, Beta distributions- Definitions, properties and applications, Normal distribution- Properties and applications.

Examples:

1.Normal Distribution: Widely used for modelling errors, measurement Uncertainties and natural phenomena.

2.Exponential Distribution: Useful for modelling waiting times, lifetimes, and Arrival processes.

3.Uniform Distribution: Applicable for random number generation and 3odelling Processes with equal probabilities within a range.

4. Gamma Distribution: Versatile distribution for modelling waiting times, Survival data, and skewed distributions.

Exercise:

1. **Area Under the Curve**:

a) Provide students with different shapes (rectangles, triangles, trapezoids) and ask them to find the area.

b) Explain the concept of probability as an area under a curve.
c) Introduce the idea of a probability density function (PDF) as a curve whose total area is 1.

2. Normal Distribution Exploration:

a) Provide students with data sets that follow a normal distribution (e.g., heights, weights, IQ scores).
b) Calculate the mean and standard deviation.
c) Use graphing calculators or statistical software to plot the data and overlay a normal curve.
d) Discuss the characteristics of the normal curve (bell shape, symmetry).

Text Books:

1. Gupta. S.C. &Kapoor,V.K. (2023) . Fundamentals of Mathematical Statistics, Sultan Chand & Sons Pvt. Ltd. New Delhi.

References:

1. Bansilal and Arora (1989). New Mathematical Statistics, Satya Prakashan, New Delhi.
2. Goon A.M., Gupta M.K. and Dasgupta B. (2002): Fundamentals of Statistics, Vol. I & II, 8th Edn. The World Press, Kolkata.
3. Mukhopadhyay, P. (2015). Mathematical Statistics.Publisher: BOOKS AND ALLIED (1 January 2016).

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23STMAL236 : Probability Foundations for Artificial Intelligence

Major 3 B.Sc. Honours (Artificial intelligence) Semester :III
Time: 3 hours Maximum Marks: 70

Section – A

Answer the following 5 x 4 M = 20Marks

1.(a) In a town, it's rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability 1/2, and given that it is not rainy, there will be heavy traffic with probability 1/4. If it's rainy and there is heavy traffic, I arrive late for work with probability 1/2 on the other hand, the probability of being late is reduced to 1/8 if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25. You pick a random day. Given that I arrived late at work, Find the probability that it rained that day.(CO2-K3)

OR

(b) Define and differentiate between classical, statistical, and axiomatic approaches to probability. .(CO1-K3)

2.(a) A random process gives measurements between 0 and 1 with p.d.f. :

$$f(x)=\begin{cases}12x^3-21x^2+10x, & 0\le x\le 1\\ 0 \quad , & elsewhere\end{cases}$$. Find $P\left(X\le\frac{1}{2}\right)$.(CO3-K3)

OR

(b) Explain the distribution function and its properties. .(CO3-K3)

3.(a) Describe the key properties of the moment generating function. .(CO4-K3)

OR

(b) A building contractor is considering bidding on one of two contracts for new downtown buildings A and B. It has been estimated that a profit of 2,00,000 would be made on building A. Bidding costs for the contractor on building A would be 10,000. On building B, the estimated profit is 5,00,000 and bidding costs would be 20,000. The probability of being awarded the contract is 2/5 on building A and 1/5 on building B. (Assume bidding costs are incurred only in case the contract is not obtained.) Find the contractor's expectation for building A and Building B. .(CO4-K3)

4.(a) Explain the Memoryless property of Geometric distribution. .(CO5-K3)

OR

(b) Let X is the number of times one must throw a die until the outcome 1 has occurred 4 times then find E(X) + V(X). .(CO5-K3)

5.(a) Describe the key properties of Uniform distribution. .(CO5-K2)

OR

(b) Describe the key properties of Exponential and Gamma distributions. .(CO5-K2)

Section B

Answer the following Questions. 5 x 10M = 50Marks

6. (a) In a bolt factory machines A, B and C manufactured respectively 25%, 35% and 40% of the total. Of their output 5, 4, 2 per cent, respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Which machine is produced more defectives? .(CO2-K3)

(OR)

(b) A, B and C are three urns which contain 2 white, 1 black; 3 white, 2 black and 2 white 2 black balls respectively. One ball is drawn from urn A and put into the urn B. Then a ball is drawn from urn B and put into urn C. Then a ball is drawn from urn C. Find the chance that the ball drawn is white. .(CO2-K3)

7.(a) A random variable has the following probability distribution

x	0	1	2	3	4	5	6	7	8
P(X= x)	a	3a	5a	7a	9a	11a	13a	15a	17a

Determine (i) 'a' (ii) Find P(X<3), P(X $\geq$ 3) and P(0<X<5) (iii) Construct the distribution function of X. .(CO3-K3)

(OR)

(b) The diameter of an electric cable, say X, is assumed to be a continuous random variable with p.d.f. $f(x) = 6x(1-x),\ 0 \leq x \leq 1$ ..(CO3-K3)
Check that f(x) is p.d.f. Determine a number b such that P(X <b) = P(X >b).(Co-1, L-3)

8.(a) A symmetric die is thrown 720 times. Use Chebyshev's inequality to find the lower bound for the probability of getting 100 to 140 sixes. .(CO4-K3)

(OR)

(b) Let X be a random variable with moment generating function
$M_X(t) = \frac{1}{12} + \frac{1}{6}e^t + \frac{1}{3}e^{2t} + \frac{1}{4}e^{-t} + \frac{1}{6}e^{-2t},\ t \in ¡$. Then Find $8E(x)$..(CO4-K3)

9.(a) In a market region half of the households is known to use a particular brand of detergent powder. In a household's survey, sample of 10 households are allotted to each investigator and 2,048 investigators are appointed for the survey. How many investigators are likely to report :
(i) 3 users (ii) not more than 3 users, and (c) at least 4 users . (CO5-K3)

(OR)

(b) Let X and Y be independent Poisson variates. The variance of is 9 and
$P(X = 3 \mid X + Y = 6) = \frac{5}{54}$.Find the mean of X. (CO5-K3)

10.(a) In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution? . (CO5 -K4)

(OR)

(b) Let X and Y have joint p.d.f. : (CO5-K4)

$$g(x, y) = \begin{cases} \frac{e^{-(x+y)}x^3y^4}{\Gamma 4 \Gamma 5}; & x > 0,\ y > 0 \\ 0, & elsewhere \end{cases}$$ Find $E(U)$ & $E[U - E(U)]^2$, where $U = \frac{X}{X+Y}$

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