PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
Siddhartha Nagar, Vijayawada–520010
Re-accredited at 'A+' by the NAAC

Course Code				**23MAMAL235**			
Title of the Course				**Linear Algebra & Matrices**			
Offered to:				**B.Sc Hons (Data Science, Data Analytics)**			
L	**5**	**T**	**0**	**P**	**0**	**C**	**4**
Year of Introduction:	**2024-25**			**Semester:**			**3**
Course Category:	**MAJOR**			**Course Relates to:**	**GLOBAL**		
Year of Revision:				**Percentage:**	**NA**		
Type of the Course:				**SKILL DEVELOPMENT**			
Crosscutting Issues of the Course :				**NA**			
Pre-requisites, if any				**Basics of Mathematics**			

Course Description:

Linear algebra is a branch of mathematics that studies systems of linear equations and properties of matrices. This course covers vectors, vector spaces,matrices, system of linear equations, determinants, eigen values and eigen vectors, and linear transformations.

Course Aims and Objectives:

S. NO	**COURSE OBJECTIVES**
1	Understand the concept of Vector Space and study its properties. Identifying Linear independence and independence of vectors.
2	Apply knowledge to identify Basis of a Vector Space and dimension of it.
3	Understand the Vector Space Homomorphism, linaer transformation and its properties, facilitating a deeper understanding of range and null space of a linear transformation.
4	Apply knowledge to determine echelon form of a matrix, rank of a matrix and reduction of normal form.
5	Apply knowledge to identify and characterize eigen values and eigen vectors.

Course Outcomes

At the end of the course, the student will be able to…

CO NO	COURSE OUTCOME	BTL	PO	PSO
CO1	Explain concepts of vector space and its properties.	**K2**	**5**	**1**
CO2	Compute basis and dimension of vector space.	**K3**	**5**	**1**
CO3	Identify range and null space of a linear transformation.	**K2**	**5**	**1**
CO4	Compute rank and inverse of a matrix.	**K3**	**6**	**1**
CO5	Compute eigen values and eigen vectors of a matrix.	**K3**	**6**	**1**

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO MATRIX									
CO NO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
CO1		**3**						**2**	
CO2	**2**								**3**
CO3		**3**							**2**
CO4	**2**								**3**
CO5		**1**							**3**

Course Structure:

UNIT-I : Vector Spaces

Vector space definition – general properties of Vector space, Subspace definition – related problems, Linear sum of two subspaces, linear combination of vectors and linear span of a set – related problems, Linear dependence of vectors definition - related problems, Linear independence of vectors definition -related problems.

Description: This unit familiarizes the students, the concepts of Vector Space, Subspace and linearly dependent & independent vectors.

Examples/Applications/Case Studies:

1. Explain the definition of a Vector space with an example.
2. Examine the following vectors are linearly dependent or linearly independent (1,2,0) , (0,3,1), (-1,0,1).

Exercises:

1. Prove that the set $\{(a_1, a_2): a_1, a_2 \in \mathbb{R}\}$ is a vector space over $\mathbb{R}$ w.r.t. the operations of addition and scalar multiplication defined as $(a_1, a_2) + (b_1, b_2) = (a_1 + b_{1,} a_2 + b_2)$ and c$(a_1, a_2) = (ca_1, ca_2)$, where $(a_1, a_2), (b_1, b_2) \in V$ and c $\in \mathbb{R}$.
2. Prove that W=$\{(a_1, a_2, 0): a_1, a_2 \in F\}$ is a subspace of $V_3(F)$, where F is a field.

Web Resources:

1. **Online Math Notes - Vector Spaces:** https://sgfin.github.io/files/notes/NYU_Optimization_2017.pdf

UNIT-II: Basis and Dimension

Basis of a vector space – definition. Basis existence , Basis extension theorems, Dimension of a vector space - related problems, Dimension of a subspace theorems- related problems, Quotient space – definition.

Description: A subset S of a vector space V(F) is said to be a basis of V(F), if (i) S is a linearly independent set.(ii) S spans V, L(S)=V. In mathematics the dimension of a vector space V is the cardinality (i.e., the number of vectors) of a basis of V over its base field.

Examples/Applications/Case Studies:

1. Show that the vectors (1,2,1) , (2,1,0) , (1,-1,2) form a basis for $R^{3.}$
2. Define the quotient space.

Exercises:

1. Find the dimension of given subspace.
2. Prove that the set {(1,0,0) , (1,1,0) , (1,1,1) , (0,1,0)}spans the vector space R^3 (R).

Web Resources:

1. Basis and Dimension problems and solutions

https://people.tamu.edu/~yvorobets//MATH304-2011C/Lect2-06web.pdf

UNIT-III: Linear Transformations

Vector space homomorphism – definitions, Linear transformation, Properties of L.T., Sum of linear transformations, scalar multiplication of L.T., product of linear transformations, Algebra of linear operators - related problems, Range & Null space of a L.T. – Definitions, related problems, Rank nullity theorem – related problems.

Description: A linear transformation is a function from one vector space to another that respects the underlying (linear) structure of each vector space. A linear transformation is also known as a linear operator or map.

Examples/Applications/Case Studies:

1. Define range and null space of a linear transformation.
2. Show that the given mapping is a linear transformation.

Exercises:

1. Describe explicitly the linear transformation using given conditions.
2. Test whether the given mapping is a linear transformation or not.

Web Resources:

1. **Online Math Notes –Linear Transformations**:
https://mandal.ku.edu/math290/m290NotesChSIX.pdf

UNIT-IV: MATRICES – I

Fundamentals of Matrices, Elementary matrix operations & elementary matrices, Rank of a matrix – definition, related problems, Echelon form of a matrix, reduction to normal form, Inverse of a matrix – related problems only.

Description: Elementary matrix operations are fundamental tools in linear algebra used for solving systems of linear equations, finding matrix inverses.

Examples/Applications/Case Studies:

1. Using elementary row operations to find the rank of a given matrix.
2. Find inverse of the matrix.

Exercises:

1. Solve upper triangular and lower triangular matrices and also find its rank.
2. Reduce the given matrix into normal form.

Web Resources:

Online Math Notes – Matrices – I :

1. chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://ocw.mit.edu/courses/18-06sc-linear-algebra-fall-2011/1999c9f4accdbef05571a1014438f8dd_MIT18_06SCF11_Ses2.8sum.pdf

UNIT-V: MATRICES – II

System of linear equations – homogeneous linear equations – related problems, System of linear equations – non homogeneous linear equations – related problems, Eigen values & Eigen vectors of a matrix – definitions& related problems, Cayley - Hamilton theorem statement only, related problems.

Description: Eigen values and eigenvectors are fundamental concepts in linear algebra with applications across many fields, including physics, engineering, computer science, and data analysis.

Examples/Applications/Case Studies:

1. Solve the given system of homogeneous linear equations.
2. Solve the given system of non homogeneous linear equations.
3. Find the eigen values and eigen vectors.

Exercises: Investigate the eigen values from the given system of equations.

Web Resources:

1. **Online web notes: Matrices – II .**
   chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://ocw.mit.edu/courses/18-06sc-linear-algebra-fall-2011/1999c9f4accdbef05571a1014438f8dd_MIT18_06SCF11_Ses2.8sum.pdf
2. YouTube videos by AVM Math hub (Student connect):
   https://www.youtube.com/watch?v=5bsaKvzTyyo&t=905s

Text Books:

1. Venkateswara Rao V & Krishna Murthy N. (2006). *A textbook of Mathematics for B.A/B.Sc - Vo.l- III* (2nd Edition). S – Chand.

Reference Books:

1. Dr Anjaneyulu A. (2006). *A textbook of Mathematics for B.A / B.Sc - Vo.l- III* (3rd Edition). Deepthi Publications.
2. Sharma J.N & Vasistha A. R. (2010). *A text book ofLinear Algebra (42nd*Edition). Krishna Prakashan Mandir.

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Course Code/**Title of the Course**: 23MA**MAL235: Linear Algebra & Matrices**

Offered to: B.Sc HONS (DATA SCIENCE & DATA ANALYTICS).

Time: 3Hrs **Max Marks:70M**

SECTION – A

Answer the following Questions **5 x 4 = 20M**

1 .(a) The set W of ordered triads (x, y, 0), where x, y ϵ F is a subspace of $V_3(F)$. (CO1, K1)

(OR)

(b)Show that the system of vectors (1,3,2), (1,-7,-8), (2,1,-1) of $V_3(R)$ is L.D. (CO1, K1)

(OR)

2. (a) Define Basis of a vector space. Give an example. (CO2, K2)

(OR)

(b) Show that the vectors $\{(1,2,1),(2,1,0),(1,-1,2)\}$ *forms a basis of* R^3 (CO2, K2)

3. (a) Describe explicitly the linear transformation T: $R^2 \to R^2$ such that T(2, 3) = (4, 5) and T(1, 0) = (0,0). (CO3, K3)

(OR)

(b) Define range and null space of a linear transformation. (CO3, K3)

4. (a) Find the rank of the matrix $\begin{bmatrix} -1 & 2 & 0 \\ 3 & 7 & 1 \\ 5 & 9 & 3 \end{bmatrix}$. (CO4, K3)

(OR)

(b) Reduce the matrix into normal form and find the rank of matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$.

(CO4, K3)

5. (a) Find the eigen roots of A = $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ (CO5, K2)

(OR)

(b) State Cayley – Hamilton Theorem. (CO5, K2)

SECTION – B

Answer the following Questions **5 x 10 = 50M**

6. (a) Prove that the set Vn of all n-tuples over a field F is a vector space w.r.t. addition of n-tuples and multiplication of n-tuple by a scalar. (CO1, K1)

(OR)

(b) Express the vector $\alpha = (1,-2,5)$ as a linear combination of the vectors $e_1 = (1,1,1), e_2 = (1,2,3)\, and\, e_3 = (2,-1,1)$. (CO1, K1)

7. (a) State and prove Basis existence theorem. (CO2, K2)

(OR)

b) Let W1 and W2 be two subspaces of R^4 given by $W1 = \{(a,b,c,d) : b - 2c + d = 0\}$, $W2 = \{(a,b,c,d) : a = d, b = 2c\}$. Find the basis and dimension of i) W_1 ii) W_2. (CO2, K2)

(OR)

8. (a) State and prove Rank nullity theorem. (CO3, K3)

(OR)

(b) Find the null space, range, rank and nullity of the transformation $T : R^2 \to R^3$ defined by

T(x,y) = (x+y, x-y, y). (CO3, K3)

9. (a)Find the inverse of the Matrix $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ (CO4, K5)

(OR)

(b) Find the Rank of the Matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ (CO4, K5)

10. (a)Investigate for what values of λ, μ the simultaneous equations $x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu$ i) No solution ii) Unique solution iii) Infinite no. of solutions. (CO5, K4)

(OR)

(b) If $A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$ verify Cayley – Hamilton theorem and hence find A^{-1}. (CO5, K4)

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