PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
Siddhartha Nagar, Vijayawada–520010
Re-accredited at 'A+' by the NAAC

Course Code				**23MAMAL231**			
Title of the Course				**GROUP THEORY**			
Offered to:				**B.Sc Honours Mathematics**			
L	**5**	**T**	**0**	**P**	**0**	**C**	**4**
Year of Introduction:		**2024-25**		**Semester:**		**3**	
Course Category:		**MAJOR**		**Course Relates to:**		**GLOBAL**	
Year of Revision:		**NA**		**Percentage:**		**NA**	
Type of the Course:				**SKILL DEVELOPMENT**			
Crosscutting Issues of the Course :				**NA**			
Pre-requisites, if any				**Basics of mathematics**			

Course Description:

Abstract algebra is a branch of mathematics that studies algebraic structures such as groups, subgroups, Normal Subgroups, Homomorphisms, Permutations Groups & Cyclic groups and more abstract constructs. Unlike elementary algebra, which primarily deals with manipulating symbols and solving equations, abstract algebra focuses on understanding the fundamental properties and relationships between these algebraic structures.

Course Aims and Objectives:

S.NO	COURSE OBJECTIVES
1	Classify different types of groups and study their properties. Groups are fundamental algebraic structures that arise in various mathematical contexts, and understanding their classifications helps in identifying common structures and relationships among different mathematical objects.
2	Identifying and studying subgroups, mathematicians can understand the internal organization of a group, including its symmetries and algebraic properties.
3	Understand the relationships between different groups and their representations, facilitating a deeper understanding of symmetry, algebraic properties, and abstract structures.
4	Show relationships between objects, analyzing kernels and images, facilitating factorization and decomposition of algebraic structures, supporting representation theory and connecting algebraic concepts with geometry.
5	Apply knowledge to identify and characterize various algebraic properties within the permutation groups.

Course Outcomes

At the end of the course, the student will be able to…

CO NO	COURSE OUTCOME	BTL	PO	PSO
CO1	Understand concepts of groups and its properties	**K2**	**1**	**1**
CO2	Define subgroups and determine the given subsets of a group are sub groups.	**K2**	**2**	**2**
CO3	Explain the significance of cosets, normal subgroups and factor groups	**K5**	**6**	**1**
CO4	Explain group homomorphisms and isomorphisms	**K5**	**7**	**1**
CO5	Find cycles of a given permutations and understand the properties of cyclic groups.	**K5**	**2**	**2**

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO MATRIX									
CO NO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
CO1	**1**							**1**	
CO2		**2**							**2**
CO3		**3**						**2**	
CO4	**3**							**2**	
CO5		**3**							**2**

Course Structure:

UNIT-I : GROUPS

Binary Operation, Semi group, Algebraic Structure, Monoid, Cancellation laws, Group definition, Abelian group, Elementary Properties, Finite and Infinite groups with examples, Order of a group with examples, Addition modulo m – Definition – theorem – Problems, Multiplication Modulo P – definition- {1, 2, 3,…….p-1} where P is a prime number is a group – theorem – Problems, Order of an element of a group – Definition – Theorems.

Description: This unit familiarizes the students, the concept of Group. Groups are fundamental structures that capture the essence of symmetry and transformation. A group consists of a set of elements along with a single binary operation that combines any two elements to produce another element in the group.

Examples/Applications/Case Studies:

1. Explain axioms of a group, and find whether the given set is satisfying the properties or not.
2. Distinguish the given set is a Group or Abelian Group.
3. Extend all possible order of elements in the given set.

Exercises:

1. Show that if every element of the group G is its own inverse, then G is abelian.

2. Prove that the given set is abelian group.
3. Find the order of any element of the group.

Web Resources:

1. **Online Math Notes - Groups:** chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://www.jmilne.org/math/CourseNotes/GT.pdf

UNIT-II: SUB GROUPS

Complex definition, Multiplication of two complexes, Inverse of a complex, subgroup definition, Identity and Inverse of a subgroup, Criterion for a complex to be a subgroup, Criterion for the product of two subgroups to be a subgroup, Union and Intersection of subgroups, Cosets Definition – Properties of cosets, Index of a subgroups of a finite groups, Lagrange's Theorem.

Description: A subgroup is a set that is a subset of a larger group and that itself forms a group with the same operation as the larger group. Understanding subgroups is crucial in various areas of mathematics because they often reveal underlying structures and symmetries within a larger group.

Examples/Applications/Case Studies:

1. Find how many subgroups of order 5.
2. Find the number of Proper subgroups of the given set (Z6).
3. How to find two left or right cosets of a subgroup is disjoint or identical.

Exercises:

1. Show that Intersection of two subgroups is again a subgroup.
2. How to show a bijection between any two left cosets in a subgroup of a group.

Web Resources:

Subgroups problems and solutions

https://byjus.com/maths/subgroups/

UNIT-III: NORMAL SUBGROUPS

Definition of a normal subgroup, Proper and improper normal subgroups, Intersection of two normal subgroups, Subgroup of index 2 is a normal subgroup, Simple group, Quotient group, Criteria for the existence of a Quotient group.

Description: Normal subgroups are a special type of subgroup with key properties that play a central role in group theory. Normal subgroups are fundamental to many aspects of group theory, providing a way to study and understand the structure of groups through quotient groups and contributing to the broader theory of homomorphisms and group actions.

Examples/Applications/Case Studies:

1. Find the difference between trivial subgroup and whole group.
2. How to find the centre of the group.
3. Show that In an abelian group, every subgroup is normal.

Exercises:

1. Prove that every finite group of order 72 is not a simple group.
2. Prove that Every Quotient Group of an Abelian Group is Abelian.

Web Resources:

Online Math Notes – Normal Subgroups : chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://egyankosh.ac.in/bitstream/123456789/71714/1/Unit-6.pdf

UNIT-IV: HOMOMORPHISM

Definition of a Homomorphism, Image of a Homomorphism, Properties of a Homomorphism, Isomorphism, Automorphism definitions and elementary properties, Kernel of a homomorphism, Fundamental theorem on homomorphism of groups and Applications, Inner automorphism, outer automorphism.

Description: a homomorphism is a fundamental concept that describes a structure-preserving map between two algebraic structures. Here's a detailed description of homomorphisms, focusing on their role and properties in group theory.

Examples/Applications/Case Studies:

1. Identifying when two groups are structurally the same (isomorphic). This helps in classifying and studying groups up to isomorphism.
2. Show that the given function is homomorphism then G is abelian.

Exercises:

1. Show that the function f is homomorphism if and only if G is commutative.
2. Prove that the set of all automorphisms of a group G forms a group with respect to composition of mapping.

Web Resources:

Online Math Notes – Group Theory: extension://efaidnbmnnnibpcajpcglclefindmkaj/https://people.bath.ac.uk/gt223/MA30237/lnotes.pdf

UNIT-V: PERMUTATIONS AND CYCLIC GROUPS

Definition of a permutation group, Equal permutations, Permutation multiplications, Order of a permutation, Inverse of a permutation, Orbits and cycles of permutation, Transposition, even and odd permutations – Theorem – Related Problems, Cayley's theorem – Related Problems, Definition of a cyclic group – Properties of Cyclic group, Standard theorems on cyclic groups – related problems.

Description: Permutation groups are fundamental mathematical structures within the realm of group theory, a branch of abstract algebra. Cyclic groups are a fundamental concept in abstract algebra, specifically within the broader field of group theory. They are named "cyclic" because they are generated by a single element, akin to how rotations around a circle produce cyclic patterns.

Examples/Applications/Case Studies:

1. Show that the given permutations are even or odd.
2. Evaluate the product of two permutations.
3. Show that the given set is a cyclic group with one of the element as a generator.

Exercises:

1. Show that every cycle of length greater than 2 can be expressed as a product of transpositions.
2. Show that the given set is a cyclic group with the given multiplication modulo and also find the number of generators.

Web Resources:

Online web notes: Permutation Groups.
https://math.umd.edu/~immortal/MATH403/lecturenotes/ch5.pdf

Text Books :

1. Venkateswara Rao V, Sharma B.V.S.S & Anjaneya Sastry S. (2015). *A textbook of Mathematics (Abstract Algebra) - Vo.l- I* (2nd Edition). S – Chand.

Reference Books:

1. Dr Anjaneyulu A. (2015). *A textbook of Mathematics (Abstract Algebra) - Vol- I* (2nd Edition). Deepthi Publications.
2. Khanna M.L. (2012). *Modern Algebra (*20th Edition). Jai Prakash Nath & Co.

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Course code/Title of the course: 23MAMAL231 :Group theory
Offered to: B.Sc HONS (MATHEMATICS)
Time: 3Hrs **Max Marks:70M**

SECTION – A

Answer the following Questions **5 x 4 = 20M**

1. (a) Show that Identity element is unique. (CO1, K3)

(OR)

(b) Show that in a group G for $a, b \in G, (ab)^2 = a^2b^2 \Leftrightarrow G$ is abelian. (CO1, K3)

2. (a) If H_1, H_2 are two subgroups of a group G, then show that $H_1 \cap H_2$ is also a subgroup of G. (CO2, K2)

(OR)

(b) If H is a subgroup of a group G and $a \in G$, then show that $a \in H \Rightarrow aH = H$ (CO2, K2)

3. (a) If N, M are normal subgroups of G, then NM is also a normal subgroup of G. (CO3, K2)

(OR)

(b) Prove that every group of prime order is simple. (CO3, K3)

4. (a) Prove that every homomorphic image of an abelian group is abelian. (CO4, K3)

(OR)

(b) If for a group G, $f : G \to G$ is given by $f(x) = x^2, \forall x \in G$ is a homomorphism, prove that G is abelian. (CO4, K2)

5. (a) Examine whether $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 3 & 1 & 8 & 5 & 6 & 2 & 4 \end{pmatrix}$ is even or odd. (CO5, K5)

(OR)

(b) Find the number of generators of a cyclic group of order 15. (CO5, K5)

SECTION – B

Answer the following Questions **5 x 10 = 50M**

6. (a) Prove that the set of nth roots of unity under multiplication form a finite group. (CO1, K1)

(OR)

(b) Show that set Q+ of all positive rational numbers forms an abelian group under the composition defined by "o" such that $aob = \frac{ab}{3}, for\ a, b \in Q$ (CO1, K3)

7. (a) Prove that H is a non – empty complex of a group G. The necessary and sufficient condition for H to be subgroup of G is $a.b \in H \Rightarrow ab^{-1} \in H$ where b^{-1} the inverse of b in G.

(CO2, K5)

(OR)

b) State and Prove Lagrange's Theorem. (CO2, K5)

8. (a) Prove that A subgroup H of a group G is a normal subgroup of G iff each left coset of H in G is a right coset of H in G. (CO3, K3)

(OR)

(b) State and Prove Quotient Group Theorem. (CO3, K3)

9. (a) The necessary and sufficient condition for a homomorphism f of a group G onto a group G' with kernel K to be an isomorphism of G into G' is that K = {e}. (CO4, K5)

(OR)

(b) State and Prove Fundamental Theorem on Homomorphism of Groups. (CO4, K5)

10. (a) If a cyclic group G is generated by an element a of order n, then a^m is a generator of G iff $(m,n)=1$. (CO5, K3)

(OR)

(b) If $f=(1\quad 2\quad 3\quad 4\quad 5\quad 8\quad 7\quad 6)$, $g=(4\quad 1\quad 5\quad 6\quad 7\quad 3\quad 2\quad 8)$ are cyclic permutations, then show that $(fg)^{-1}=g^{-1}f^{-1}$. (CO5, K3)

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