



PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
 Siddhartha Nagar, Vijayawada-520010
Re-accredited at 'A+' by the NAAC

Course Code				23MAMAL233			
Title of the Course				Laplace transforms			
Offered to:				B sc. Honours (Mathematics)			
L	5	T	0	P	0	C	4
Year of Introduction:		2024-25		Semester:			3
Course Category:		Major		Course Relates to:		GLOBAL	
Year of Revision:		NA		Percentage:		NA	
Type of the Course:				SKILL DEVELOPMENT			
Crosscutting Issues of the Course :				NA			
Pre-requisites, if any				Basics of Differential Equations with Course code:23MAMIL121			

Course Description:

An overview of the course content and objectives.

Laplace transform is a fundamental tool in integral calculus. It is used to solve various types of differential equations, difference equations, integral equations etc. which arise naturally in engineering and basic sciences.

Course Aims and Objectives:

S.N O	COURSE OBJECTIVES
1	Knowledge in fundamental concepts of Laplace transforms of a function
2	Ability to develop necessary skills to recognize the properties of Laplace transforms their applications
3	Understand Laplace transforms of various functions
4	Ability to understand the basic concepts of inverse transforms of a functions
5	Competence in concepts of convolution theorem, Heaviside expansion and their applications

Course Outcomes

At the end of the course, the student will be able to...

CO NO	COURSE OUTCOME	BT L	P O	PS O
CO1	Understand the definition and properties of Laplace transformations	K2	7	1
CO2	Get an idea about first and second shifting theorems and change of scale property	K5	6	2
CO3	Analyse the Laplace transform of various functions	K5	1	1
CO4	Know the reverse transformation of Laplace and its properties	K4	2	2
CO5	Get the knowledge of application of convolution theorem	K3	3	2

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO MATRIX									
CO NO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PSO1	PSO2
CO1	2							1	
CO2		3							2
CO3	2							3	
CO4		1							1
CO5	3								3

Use the codes 3,2,1 for High, Moderate and Low correlation Between CO-PO-PSO respectively

Course Structure:

Unit-1

Laplace transforms-1

15 hrs

Definition of Laplace transform- linearity property-piecewise continuous functions-Existence of Laplace transforms- functions of exponential order and of class A, First shifting theorem.

Description: This unit familiarizes the students, the concept of Laplace transforms. The Laplace transform is used to solve differential equations.

Examples/Applications/Case Studies:

1. Explain how to apply laplace transforms for given functions.
2. Extend the different properties of laplace transforms can be applied or not
3. Combining some of these simple laplace transforms with the properties of the laplace transforms

Exercises:

1. Find $L(\sin 2t \cos 3t)$
2. Find the laplace transform of the function $F(t) = 4, 0 < t < 1$
 $3, t > 1.$
3. Evaluate $L(\sin \hat{t} \cos \hat{t})$

Web Resources:

Online Math Notes – laplace transforms -I:

https://sist.sathyabama.ac.in/sist_coursematerial/uploads/SMT1401.pdf

Unit-2: Laplace Transforms-II

Second Shifting Theorem, Change of Scale Property, Laplace transform of the derivative of $f(t)$, Initial value theorem and Final value theorem.

Description: The second shift theorem is similar to the first except that, in this case. It is the time variable that is shifted not the t - variable

Examples/Applications/Case Studies:

1. If $L(F(t)) = \frac{9p^2 - 12p + 15}{(p-1)^3}$ then find $L(F(3t))$ using change of scale property
2. Evaluate $L(G(t))$ where $G(t) = \begin{cases} \cos(t - \frac{\pi}{3}), t > \frac{\pi}{3} \\ 0, t < \frac{\pi}{3} \end{cases}$
3. State and prove initial value theorem

Exercises:

1. Find the laplace transform of $\cos at$ using the theorem on transforms of derivatives
2. Find $L(\sin^2(at))$ by using change of scale property

Web Resources:

YouTube videos :

<https://www.youtube.com/watch?v=C0GPZZ15Shk>

Unit-3:Laplace Transforms-III

Laplace transform of Integrals and related problems– Multiplication by t, Multiplication by t^n and related problems- division by t and related problems- Evaluation of integrals by laplace transforms

Description: The laplace transform's key property is that it converts differentiation and integration in the time domain into multiplication and division in the laplace domain. The laplace transform is an integral transform that can be used to evaluate integrals, it converts a function of real variable into a function of a complex variable.

Examples/Applications/Case Studies:

1. Find $L(\frac{e^{-a} - e^{-b}}{t})$
2. Evaluate $L(t \cos 3t)$
3. Show that $\int_0^{\infty} t e^{-3t} \sin t dt = \frac{3}{50}$

Exercises:

1. Find the laplace transform of $\frac{1-\cos}{t^2}$
2. Find $L((t^2-3t+2)\sin 3t)$
3. Evaluate $\int_0^{\infty} t e^{-3t} dt$

Web Resources:

Online Math Notes- laplace transform-III: https://mathalino.com/reviewer/advance-engineering-mathematics/evaluation-integrals#google_vignette

Unit -4: Inverse Laplace transforms –I

Definition of inverse Laplace transform – linearity property - First shifting Theorem-Second Shifting Theorem- Change of Scale Property- use of partial fractions – examples

Description: The inverse laplace transform is used to find the original function from its laplace transform. It's a powerful tool for solving non homogeneous linear differential equations which are equations where the solution to the derivative is not zero

Examples/Applications/Case Studies:

1. Find the inverse laplace transform of $\frac{2p-5}{p^2-9}$
2. Evaluate inverse laplace transform of $\left(\frac{p+1}{p^2+6p+25}\right)$

Exercises:

1. Find the inverse laplace transform of $\frac{e^{-3p}}{(p-2)^2}$
2. Find the inverse laplace transforms of $\frac{3p+1}{p^2-2p-3}$ by partial fractions

Web Resources:

1. **Online Math Notes – inverse laplace transform –I:**
<https://www.vedantu.com/maths/inverse-laplace-transform>

Unit -5: Inverse Laplace transforms –II

Inverse laplace transforms of derivatives- inverse laplace transforms of Integrals- multiplication by powers of 'p'-Division by powers of 'p'-convolution Definition- Convolution theorem –Proof and Applications- Heaviside's expansion theorem and its Applications

Description: The convolution theorem for laplace transforms states that taking the convolution of two functions and then taking the laplace transform is the same as taking the laplace transform of each function separately and then multiplying the two laplace transform together

Examples/Applications/Case Studies:

1. Find $L^{-1}\left(\frac{1}{p}\log\left(\frac{p+2}{P+1}\right)\right)$ by division by 'p'
2. Find $L^{-1}\left(\frac{P+3}{(p^2+6p+13)^2}\right)$

Exercises:

1. Using Heaviside's expansion formula, Find $L^{-1} \left(\frac{3p+1}{(p-1)(p^2+1)} \right)$
2. Using convolution theorem, $L^{-1} \left(\frac{1}{p(p+1)(p+2)} \right)$

Web Resources:**Online web notes -Inverse laplace transform:**

[https://math.libretexts.org/Bookshelves/Differential_Equations/Introduction_to_Partial_Differential_Equations_\(Herman\)/09%3A_Transform_Techniques_in_Physics/9.09%3A_The_Convolution_Theorem](https://math.libretexts.org/Bookshelves/Differential_Equations/Introduction_to_Partial_Differential_Equations_(Herman)/09%3A_Transform_Techniques_in_Physics/9.09%3A_The_Convolution_Theorem)

Text Books:

1. Vashistha A.R & Dr. Guptha R.k,2017, *Integral transform*, 37th edition, Krishna prakashan media Pvt..Ltd.,meeru.

References:

- 1 .Goyal J.K & Guptha K.P, 2011, *Laplace and Fourier transform*, 24 th edition, Pragathi Prakashan.
2. Raisinghania .M.D, 1995, *integral transforms*, 2nd edition S-Chand & co.



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Course Code/ Title of the Course: 23MAMAL233 : LAPLACE TRANSFORMS

Offered to: B.Sc.Hons. Mathematics

Time: 3Hrs

MAX MARKS: 70M

SECTION – A

Answer the following Questions.

5 x 4 = 20 M

1.(a) Find $L(7e^{2t} + 9e^{-2t} + 5\cos t + 7t^3 + 5\sin 3t + 2)$ (CO1, K5)

(OR)

1.(b) Find the laplace transform of the function $F(t) = 4, 0 < t < 1$

$3, t > 1.$ (CO1, K5)

2. (a) If $L[F(t)] = \frac{p^2 - p + 1}{(2p + 1)^2(p - 1)}$ then show that $L\{F(2t)\} = \frac{p^2 - 2p + 4}{4(p + 1)^2(p - 2)}$ by applying

change of scale property

(CO2, K3)

(OR)

2. (b) State and prove Second shifting theorem (CO2, K3)

3. (a) Find $L\left\{\int_0^t \frac{e^t \sin t}{t} dt\right\}$

(CO3, K2)

(OR)

3.(b) Find the laplace transform of e^{at} using the theorem on transforms of derivatives

(CO3, K2)

4.(a) Find $L^{-1}\left(\frac{3p - 4}{p^2 - 4p + 8}\right)$

(CO4, K3)

(OR)

4.(b) find the inverse laplace transform of $\frac{3}{p^2 - 3} + \frac{3p + 2}{p^3} - \frac{3p - 27}{p^2 + 9} + \frac{6 - 30\sqrt{p}}{p^4}$

(CO4, K3)

5.(a) Find $L^{-1}\left(\frac{1}{p} \log\left(\frac{p + 2}{p + 1}\right)\right)$ by division by 'p' (CO5, K2)

(OR)

5.(b) Find $L^{-1} \left(\frac{P+3}{(p^2+6p+13)^2} \right)$ (CO5, K2)

SECTION-B

Answer the following

10x5=50M

6.(a) Find $L(\sin t \cos t)$ (CO1, K1)

(OR)

6.(b) using expansion $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$ Show that $L(\sin \sqrt{t}) = \frac{\sqrt{\pi}}{2p^{3/2}} e^{-1/4p}$ (CO1, K1)

7.(a) State and prove Initial value theorem (CO2, K3)

(OR)

7.(b)(i) If $L(F(t)) = \frac{9p^2 - 12p + 15}{(p-1)^3}$ then find $L(F(3t))$ using change of scale property (CO2, K3)

(ii) Find $L(G(t))$ where $G(t) = \begin{cases} \cos(t - \frac{\pi}{3}), t > \frac{\pi}{3} \\ 0, t < \frac{\pi}{3} \end{cases}$ (CO2, K3)

8.(a) Find $L((t^2 - 3t + 2)\sin 3t)$ (CO3, K5)

(OR)

8.(b)(i) Find the laplace transform of $L\left(\frac{\sin 3t \cos t}{t}\right)$ (CO3, K5)

(ii) Find the laplace transform of $\left(\frac{\cos at - \cos bt}{t}\right)$ (CO3, K5)

9.(a) Find the inverse laplace transforms of $\frac{3p+1}{p^2-2p-3}$ by partial fractions (CO4, K3)

(OR)

9.(b) (i) Find $L^{-1}\left(\frac{e^{4-3p}}{(p+4)^{5/2}}\right)$ (ii) Find the inverse laplace of $\frac{e^{-\pi p}(p+1)}{p^2+p+1}$ (CO4, K3)

10.(a) Using Heaviside's expansion formula, Find $L^{-1} \left(\frac{3p+1}{(p-1)(p^2+1)} \right)$ (CO5, K2)

(OR)

10.(b) Using convolution theorem, $L^{-1} \left(\frac{1}{p(p+1)(p+2)} \right)$ (CO5, K2)