PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
Siddhartha Nagar, Vijayawada–520010
Re-accredited at 'A+' by the NAAC

Course Code				**23MAMAL234**			
Title of the Course				**Special functions**			
Offered to:				**B.Sc Hons (Mathematics)**			
L	**5**	**T**	**1**	**P**	**0**	**C**	**4**
Year of Introduction:	**2024-25**			**Semester:**			**3**
Course Category:	**MAJOR**			**Course Relates to:**	**GLOBAL**		
Year of Revision:	**NA**			**Percentage:**	**NA**		
Type of the Course:				**SKILL DEVELOPMENT**			
Crosscutting Issues of the Course :				**NA**			
Pre-requisites, if any				**Basics of Mathematics**			

Course Description:

The properties of special functions like Gauss hypergeometric, Legendre functions with their integral representations. Understand the concept of Bessel's function, Hermite function etc, with its properties like recurrence relations, orthogonal properties, generating functions.

Course Aims and Objectives:

S.NO	COURSE OBJECTIVES
1	Acquire the information about Beta and Gamma functions, and evaluate it in various Problems.
2	Derive Rodrigue's formula, generating function, recurrence relations and orthogonal Property of Laguerre polynomials and use them in various applications
3	Solve Hermite equation and write the Hermite Polynomial of order 'n' also find the generating function and orthogonal properties of Hermite polynomials
4	Solve Legendre equation and write the Legendre equation of first kind, also find the generating function and orthogonal properties of Legendre Polynomials
5	Solve Bessel's equation and write the Bessel's equation of first kind also find the generating function of Bessel's function.

Course Outcomes

At the end of the course, the student will be able to…

CO NO	COURSE OUTCOME	BTL	PO	PSO
CO1	Understand concepts of Beta and Gamma functions	**K3**	**6**	**1**
CO2	orthogonal Property of Laguerre polynomials and use them in various applications.	**K1**	**1**	**2**
CO3	Explain the orthogonal properties of Hermite polynomials	**K2**	**5**	**1**
CO4	Explain orthogonal properties of Legendre Polynomials	**K2**	**6**	**1**
CO5	Find generating function of Bessel's function.	**K4**	**6**	**1**

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO MATRIX									
CO NO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
CO1	**1**							**1**	
CO2		**2**							**2**
CO3		**3**						**2**	
CO4	**2**							**2**	
CO5		**3**							**2**

Course Structure:

UNIT-I :

UNIT – I: BETA AND GAMMA FUNCTIONS, CHEBYSHEV POLYNOMIALS (15hrs)

1.1 - Euler's Integrals-Beta and Gamma Functions, Elementary properties of Gamma Functions,
1.2 - Transformation of Gamma Functions, Another form of Beta Function,
1.3 - Relation between Beta and Gamma Functions.
1.4 - Chebyshev polynomials, orthogonal properties of Chebyshev polynomials
1.5 - Recurrence relations, generating functions for Chebyshev polynomials

Description: This course delves into the Beta and Gamma functions, which are fundamental in various areas of mathematics and applied sciences.

Examples/Applications/Case Studies:

 Generalized Functions: The Beta and Gamma functions are often used together in problems involving integrals that appear in generalized functions and in the study of hypergeometric functions.

Web Resource:

1. **Online Math Notes –**
2. https://web.mst.edu/~lmhall/SPFNS/spfns.pdf

UNIT –II: LAGUERRE POLYNOMIALS

2.1 - Laguerre's differential equation
2.2 - Laguerre polynomials
2.3 - Generating function
2.4 - Other forms for Laguerre polynomials
2.5 - Rodrigue's formula
2.6 - To find first few Laguerre polynomials
2.7 - Orthogonal properties for Laguerre polynomials
2.8 - Recurrence formula for Laguerre polynomials.

Description Laguerre polynomials are a sequence of orthogonal polynomials that arise in various areas of mathematical analysis and physics, particularly in the study of quantum mechanics and differential equations. They are named after the French mathematician Edmond Laguerre..

Examples/Applications/Case Studies:

Angular Momentum: They also appear in the study of angular momentum in quantum mechanics, particularly in problems involving spherical coordinates. Find the number of Proper subgroups of the given set .

Differential Equations: They are solutions to the Laguerre differential equation, which is used in various physical models and boundary value problems.

Exercises:

1. Compute $L_0(x)$, $L_0(x)L_0(x)$ and $L_1(x)L_1(x)L_1(x)$ explicitly..
2. Compute $L_2(x)L_2(x)L_2(x)$ using the recurrence relation:

Web Resources:

1. https://web.mst.edu/~lmhall/SPFNS/spfns.pdf

UNIT-III:

HERMITE POLYNOMIALS

3.1. Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials,
3.2. Generating function for Hermite polynomials.
3.3. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few hermite Polynomials.
3.4. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

Description: Hermite polynomials are a family of orthogonal polynomials that arise in probability theory, combinatorics, and the solution of differential equations. They are particularly important in quantum mechanics and statistical physics

Examples/Applications/Case Studies:

Quantum Mechanics

- **Quantum Harmonic Oscillator**:

- **Normal Distribution**.

Orthogonal Polynomials:

Exercises:

1. Verify the first few Hermite polynomials using their recurrence relation: Hn+1(x)=2xHn(x)−2nHn−1(x)H_{n+1}(x) = 2x H_n(x) - 2n H_{n-1}(x)Hn+1(x)=2xHn(x)−2nHn−1(x)
2. Using the generating function:

Find H2(x)H_2(x)H2(x) and H3(x)H_3(x)H3(x) by expanding the right-hand side up to t^3 t3t3.

Web Resources:

1. **Online Math Notes –**

1. http://www.physics.wm.edu/~finn/home/MathPhysics.pdf

UNIT-IV:

LEGENDRE'S POLYNOMIALS

4.1. Definition, Solution of Legendre's equation, Legendre polynomial of degree n,
4.2. Generating function of Legendre polynomials.

4.3. Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation (derivations not required) to show that Pn(x) is the coefficient of h^n , in the expansion of $(1 - 2xh + h^2)^{-1/2}$.
4.4. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

Description: Legendre polynomials are a class of orthogonal polynomials that arise in various areas of mathematics and physics. They are particularly significant in solving problems with spherical symmetry, such as in potential theory and quantum mechanics

Examples/Applications/Case Studies:

1. Spherical Harmonics Expansion.
2. **Hydrogen Atom**
3. Numerical Integration

Exercises:

1. Verify the recurrence relation for Legendre polynomials:

2. Derive the generating function for Legendre polynomials:.

Web Resources:

2. **Online Math Notes –** https://www.math.tamu.edu/~fnarc/psfiles/special_fun.pdf

UNIT-V: BESSEL'S EQUATION (15 hrs)

5.1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n,
5.2. Bessel's function of the second kind of order n.
5.3. Integration of Bessel's equation in series form=0,
5.4. Definition of Jn(x), recurrence formulae for Jn(x).
5.5. Generating function for Jn(x), orthogonally of Bessel function

Description:

Bessel functions are a family of solutions to Bessel's differential equation, which appears in many problems of mathematical physics, particularly those involving cylindrical symmetry. They play a crucial role in fields such as wave propagation, heat conduction, and electromagnetic theory.

Examples/Applications/Case Studies:

1. **Wave Propagation**
2. Heat Conduction

Exercises:

1. Find the series expansion of the Bessel function of the first kind Jn(x)J_n(x)Jn(x) for n=0n = 0n=0. Use Rodrigues' formula:
2. Verify the recurrence relation for Bessel functions of the first kind:.

Web Resources:

3. **Online web notes:** https://nitkkr.ac.in/docs/18-%20Series%20Solution%20and%20Special%20Functions.pdf

Text Books :

1. Dr.M.D.Raisinghania, Ordinary and Partial Differential Equations, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.

Reference Books:

1. J.N. Sharma and Dr.R.K. Gupta, Differential equations with special functions, Krishna Prakashan Mandir.
2. Shanti Narayan and Dr.P.K.Mittal, Integral Calculus, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
3. George F. Simmons, Differential Equations with Applications and Historical Notes, Tata McGraw-Hill Edition, 1994.
4. Shepley L. Ross, Differential equations, Second Edition, John Willy & sons, New York, 1974.

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CourseCode/Title of the Course : 23MA**MAL234: SPECIAL FUNCTIONS**

Offered to: B.Sc HONS (MATHEMATICS)

Time: 3Hrs **Max Marks : 70M**

SECTION – A

Answer the following Questions **5x4=20M**

1. (a). Evaluate $\int_0^\infty x^2\, e^{-x^2} dx$

(OR) (CO1, K5)

(b). P.T $\Gamma\left(\frac{3}{2}-x\right)\Gamma\left(\frac{3}{2}+x\right)=\left(\frac{1}{4}-x^2\right)\pi sec\pi x$ where -1<2x<1 (CO1, K5)

2. (a). Prove that $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(x^n e^{-x})$. (CO2, K3)

(OR)

(b). Prove that $L_n(0) = 1$. (CO2, K3)

3. (a). Prove that $H_{2n}(0)$ = (-1)$^n \frac{(2n)!}{n\,!}$ (CO3, K1)

(OR)

(b). Prove that $H_n{}^1(x) = 2nH_{n-1}(x)$ (CO3, K1)

4. (a). Prove that $P_n \ (-x) = (-1)^n P_n(x)$

(OR)

(b). Prove that $(1-x^2)P_n{}^1(-x) = (n+1)(xP_n(x) - P_{n+1}(x))$ (CO4, K3)

5. (a) Prove that $xJ_n{}^1(x) = nJ_n(x) - xJ_{n+1}(x)$ (CO5, K4)

(OR)

(b) State and prove Generating function for J_n(x). (CO5, K4)

SECTION – B

Answer the following Questions **5x10=50M**

6. **(a).** Prove that $\beta(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$. (CO1, K1)

(OR)

(b). When n is a positive integer prove that $2^n \Gamma(n+\frac{1}{2}) = 1.3.5\ldots\ldots\ldots.(2n-1)\sqrt{\Pi}$.
(CO1, K1)

7. (a). State and Prove Generating function for Laguerre polynomials. (CO2, K2)

(OR)

(b). Prove that $(n+1) L_{n+1}(x) = (2n+1-x) L_n(x) - n L_{n-1}(x)$. (CO2, K2)

8. (a). State and Prove Generating function for Hermite Polynomials (CO3, K5)

(OR)

(b). Prove that $H_n(x) = 2^n [\exp(-\frac{1}{4}\frac{d^2}{dx^2}) x^n]$. (CO3, K5)

9. (a). Prove that $(2n+1) xP_n(x) = (n+1)P_{n+1}(x) + n P_{n-1}(x)$. (CO4, K3)

(OR)

(b). State and prove Rodrigues formula for Legendre's Equation. (CO4, K3)

10. (a) Prove that $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$. (CO5, K5)

(OR)

(b). Prove that $\sqrt{(\frac{\Pi x}{2})}J_{\frac{3}{2}}(x) = \frac{1}{x}\sin x - \cos x$. (CO5, K5).

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