PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
Siddhartha Nagar, Vijayawada–520010
Re-accredited at 'A+' by the NAAC

23STMAL122: Random Variables & Mathematical Expectations

Offered to : B.Sc. Honours (Statistics)   **Course Type** : Major 4 (Th)

Year of Introduction : 2023-24   **Semester: II**   **60Hrs**   **Credits: 3**

Course Prerequisites: Student required basic knowledge in Mathematics

Course Description: This course helps the students to familiarize with the ways in which we talk about uncertainty and estimate their situations in which probability arises. Also this course aims at providing basic knowledge about mathematical expectations & generating functions.

Course Objectives:
1) To analyze data pertaining to discrete and continuous variables and to interpret the results.
2) To learn and be able to apply the properties of mathematical expectation.
3) To introduce law of large numbers and Central limit theorem (CLT)

Learning Outcomes: At the end of the course, the student will able to:
1) To acquaint with the role of statistics in dealing with the univariate random variables.
2) To learn the extension of the univariate data to bivariate data.
3) To learn the measure of randomness mathematically by using expectations.
4) To get the familiarity about the generating functions, law of large numbers and central limit theorem, further to apply in research and allied fiends.

Course Outcomes:		
Course Outcome	After completion of this course, student will able to:	**Program Outcomes Mapping**
CO 1	Understand the concept of discrete and continuous random variables with the application of random variables in real time problems.	PO1
CO 2	Learn the variance and covariance of random variables in terms of expectation	PO1
CO 3	Understand the definitions of various generating functions and learns their applications.	PO1
CO 4	Learn how apply the concepts of Weak Law of Large Numbers (WLLN) and Strong Law of Large Numbers (SLLN)	PO2
CO 5	Applying the concept of Central limit theorem in real life examples and various inequalities in expectation	PO2

CO-PO MATRIX							
23STMAL122	CO-PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7
	CO1	3						
	CO2	3						
	CO3	3						
	CO4		3					
	CO5		3					

Syllabus

Course Details

Unit	Learning Units	Lecture Hours
I	**Univarate Random Variables:** Definition, Discrete and Continuous random variables -Probability mass function and Probability density function with illustrations. Distribution function and its properties. Calculation of moments, coefficient of skewness and kurtosis for a given probability mass function(PMF) and Probability Density function (PDF)	12
II	**Mathematical Expectations:** Definition, Mathematical expectation of a random variable, Properties of expectations. Moments and covariance using mathematical expectation and their properties. Addition and Multiplication theorems on expectation of two and n variables. Variance, variance of a Linear combination of Random variables	12
III	**Generating Functions:** Definitions of Moment Generating Function (M.G.F.), Cumulant Generating Function (C.G.F), Probability Generating Function (P.G.F), Characteristic Function (c. f.) and their properties with applications.	12
IV	Weak Law of Large Numbers (WLLN) and Strong Law of Large Numbers (SLLN- Statement only) for identically and independently distributed (i. i. d.) random variables with finite variance. Markov's inequality (Statements only), Khinchin's Theorem for WLLN (Statements only).	12
V	Chebychev's and Cauchy - Schwartz inequalities and their applications. Central limit theorems, Statement of De-Movire's Laplace theorem, Lindberg – Levy CLT and its applications, Statement of Liapounoff's CLT, relationship between CLT and WLLN.	12

Text Book:

1. Fundamentals of Mathematical Statistics, 12th Edition, 2020, S. C. Gupta and V. K. Kapoor, Sultan Chand & Sons, New Delhi

Reference Books:

1. B.A/B.Sc. Second Year Statistics(2010) , Telugu Akademi, Hyderabad.
2. Mathematical Statistics with Applications, 2009, K.M.Ramachandran and Chris P.Tsokos Academic Press(Elsevier), Haryana .
3. Probability and Statistics, Volume I & II, D. Biswas, New central book Agency (P) Ltd, NewDelhi.
4. An outline of Statistical theory, Volume II,3rd Edition,2010(with corrections) A.M.Goon,M.K. Gupta, B.Dasgupta ,The World Press Pvt.Ltd., Kolakota.
5. Sanjay Arora and BansiLal:. New Mathematical Statistics, SatyaPrakashan , New Delhi.

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23STMAL122: Random Variables & Mathematical Expectations

Max. Marks : 70   **Semester II**   **Max. Time : 3Hrs**

Section - A

Answer the following questions   **5 X 4M = 20M**

1. a. Define the Random variables and state its types.   (Co-1, L-1)
    (OR)
   b. Define distribution function and state its properties   (Co-1, L-1)

2. a. Show that the mathematical expectation of the sum of two random variables is the sum of their individual expectation.   (Co-2, L-1)
    (OR)
   b. State and prove multiplication theorem on Mathematical expectation of two events.
                    (Co-2, L-1)
3. a. Define Probability Generating Function(PGF) and write its properties.   (Co-3, L-1)
    (OR)
   b. Define Characteristic Function (CF) and write its properties.   (Co-3, L-1)

4. a. Explain the concept of Weak law of large numbers(WLLN).   (Co-4, L-2)
    (OR)
   b. Explain the concept of Strong Law of Large Numbers (SLLN).   (Co-4, L-2)

5. a. State the Liapounoff's central limit theorem.   (Co-5, L-1)
    (OR)
   b. State the Lindberg – Levy's theorem and its assumptions.   (Co-5, L-1)

Section - B

Answer the following questions 5 X 10M = 50M

6. a. A random variable has the following probability distribution

x	0	1	2	3	4	5	6	7	8
P(X= x)	a	3a	5a	7a	9a	11a	13a	15a	17a

    (i) Determine 'a'
    (ii) Find P(X<3), P(X $\geq$ 3) and P(0<X<5)
    (iii) Find the distribution function of X. (Co-1, L-3)

                (OR)

b. The diameter of an electric cable, say X, is assumed to be a continuous random variable with p.d.f. $f(x) = 6x(1-x)$, $0 \leq x \leq 1$.
    i) Check that f(x) is p.d.f.,
    ii) Determine a number **b** such that P(X <**b**) = P(X >**b**). (Co-1, L-3)

7. a. State and prove the Linear combination of Random variables (Co-2, L-5)

                (OR)

    b. Given the following table:

X = x	-3	-2	-1	0	1	2	3
P(X = x)	0.05	0.1	0.3	0	0.3	0.15	0.1

    Compute (i) E(X), (ii) E(2X+3), (iii) V(X) and (iv) V(2X+3) (Co-2, L-3)

8. a. Derive the relation between cumulants in terms central moments. (Co-3, L-3)

                (OR)

    b. Prove that $\mu_r^1 = \left[\dfrac{d^r M_X(t)}{dt^r} \right]_{t=0}$ (Co-3, L-3)

9. a. Examine whether the weak law of large numbers holds for the sequence {X_k} of independent random variable defined as follows: $P(X_k = \pm 2^k) = 2^{-(2k+1)}$, $P(X_k = 0) = 1 - 2^{-2k}$ (Co-4, L-3)

                (OR)

    b. Write the statements of W.L.L.N and S.L.L.N. for the sequence of i.i.d. random variables (Co-4, L-3)

10. a. Use chebychev's inequality to determine how many times a fair coin must be tossed in order that the probability will be atleast 0.90 that the ratio of the observed number of heads to the number of tosses will lie between 0.4 and 0.6. (Co-5, L-3)

                (OR)

    b. State and prove Cauchy - Schwartz inequalities (Co-5, L-3)

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