PARVATHANENI BRAHMAYYA
SIDDHARTHA COLLEGE OF ARTS & SCIENCE
Autonomous
Siddhartha Nagar, Vijayawada–520010
Re-accredited at 'A+' by the NAAC

Course Code				**23STMAL232**			
Title of the Course				**Continuous Probability Distributions**			
Offered to: (Programme/s)				**B.Sc. Hons Statistics**			
L	**4**	**T**	**0**	**P**	**0**	**C**	**3**
Year of Introduction:		**2024-25**		**Semester:**			**3**
Course Category:		**Major**		**Course Relates to:**		Local, Regional, National, Global	
Year of Revision:		**NA**		**Percentage:**		**NA**	
Type of the Course:				Skill development			
Crosscutting Issues of the Course:				**NA**			
Pre-requisites, if any				**Probability & Random Variable**			

NA : Not Applicable

Course Description:

Continuous probability distributions explore the fundamentals of probability theory with a focus on continuous random variables. Students will learn about key concepts such as probability density functions, expected values, variance, and common distributions including rectangular, exponential, Laplace, beta, gamma, Cauchy, normal and log - normal . The course emphasizes practical applications and problem-solving techniques, providing a solid foundation for analyzing and interpreting data in various fields. By the end of the course, students will be equipped to model real-world scenarios using continuous probability distributions and apply statistical methods to draw meaningful conclusions from data.

Course Aims and Objectives:

S.NO	COURSE OBJECTIVES
1	**Define and Describe:** Clearly define and describe key concepts related to continuous probability distributions, including probability density functions and cumulative distribution functions.
2	**Identify and Use Distributions:** Identify and apply common continuous probability distributions, such as exponential, uniform, beta, gamma, and normal in various scenarios.
3	**Perform Calculations:** Accurately perform calculations involving probabilities, expected values, and variances for continuous random variables.
4	**Interpret Results:** Interpret the results of probability computations and understand their implications in real-world contexts.
5	**Apply Problem-Solving Techniques:** Utilize continuous probability distributions to solve practical problems, employing appropriate statistical methods and techniques.

Course Outcomes

At the end of the course, the student will be able to…

CO NO	COURSE OUTCOME	BTL	PO	PSO
CO1	explain the key properties of continuous probability distributions	**K2**	**PO1**	**PSO1**
CO2	compute the moments and deviations for continuous distributions	**K3**	**PO1**	**PSO1**
CO3	analyze the memoryless property and additive properties in distributions	**K4**	**PO1**	**PSO1**
CO4	derive and interpret moment generating and characteristic functions	**K4**	**PO1**	**PSO1**
CO5	apply various continuous distributions to solve real-world problems	**K3**	**PO1**	**PSO1**

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO MATRIX									
CO NO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PSO1**	**PSO2**
CO1	**2**							**2**	
CO2	**2**							**2**	
CO3	**3**							**2**	
CO4	**2**							**2**	
CO5	**3**							**2**	

Use the codes 3,2,1 for High, Moderate and Low correlation Between CO-PO-PSO respectively

Course Structure:

Unit – 1: Continuous Uniform distribution (12Hrs)

Continuous Uniform distribution – Definition, distribution function, Moments of Continuous Uniform distribution,Median of Continuous Uniform distribution, Mean deviation about Mean of Continuous Uniform distribution, Moment generating function and Characteristic function,of Continuous Uniform distribution. Measure of skewness, kurtosis and problems.

Examples/Applications/Case Studies:

1. **Random Number Generation:** Many programming languages and statistical software use the continuous uniform distribution to generate random numbers within a specified range.
2. **Simulation Studies:** In various fields, such as engineering, finance, and social sciences, the continuous uniform distribution is often used to simulate random events

or variables. For example, it can be used to simulate the arrival time of customers in a queueing system or the random errors in a measurement process.

3. **Sampling:** The continuous uniform distribution can be used to select random samples from a population, ensuring that each member of the population has an equal chance of being selected.

Exercises/Projects:

Project 1: Simulating Dice Rolls

Goal: Simulate the rolling of a fair six-sided die using the continuous uniform distribution.

Steps:

1. **Generate random numbers:** Use the runif() function in R to generate a large number of random numbers between 1 and 6.
2. **Round to integers:** Round the generated numbers to the nearest integer to simulate a die roll.
3. **Analyze results:** Calculate the frequency of each outcome, compare it to the expected frequency (1/6 for each outcome), and visualize the results using a histogram.

Project 2: Monte Carlo Simulation for Pi Estimation

Goal: Estimate the value of pi using a Monte Carlo simulation based on the continuous uniform distribution.

Steps:

1. **Generate random points:** Generate a large number of random points within a square with side length 2, centered at the origin.
2. **Calculate distances:** Calculate the distance of each point from the origin.
3. **Count points within the circle:** Determine how many of the generated points fall within a circle with radius 1, centered at the origin.
4. **Estimate pi:** Calculate the estimated value of pi using the formula: pi ≈ 4 * (number of points within the circle) / (total number of points)

Specific Resources (Web):

1. **https://univ.jeanpaulcalvi.com/Posters/ConfAuchWeb/abramovitz2.pdf**
2. **Stat Trek: https://www.youtube.com/watch?v=bPEctT-xb1o**

Unit – 2: Exponential Distribution (12Hrs)

Exponential distribution – Definition, Moments of Exponential distribution,Median of Exponential distribution, Moment generating function and Characteristic function of Exponential distribution.The first four moments are obtained through moment generating function, Memoryless property of Exponentialdistribution.Measure of skewness, kurtosis and problems.

Double Exponentialor Laplace Distribution- Definition, Moments of Double Exponential distribution,Moment generating function and Characteristic function of double Exponential distribution. The first four moments are obtained through characteristic function,Measure of skewness, kurtosis and problems.

Examples/Applications/Case Studies:

1. **Customer Service:**

   Scenario: A call center receives an average of 20 calls per hour.

   Analysis: The time between calls follows an exponential distribution. This can be used to model wait times, staffing requirements, and system performance.

2. **Equipment Failure:**

   Scenario: A machine has a mean time between failures (MTBF) of 1000 hours.

   Analysis: The time until the next failure follows an exponential distribution. This can be used to predict equipment reliability, maintenance schedules, and replacement costs.

Exercises/Projects:

1. **Queuing System Simulation**

Project Goal: Simulate a queuing system (e.g., a call center, a grocery store checkout) using the exponential distribution to model inter-arrival and service times.

Steps:

1. **Define parameters:** Determine the average arrival rate and service rate for the system.
2. **Generate inter-arrival and service times:** Use the exponential distribution to generate random inter-arrival and service times for customers.
3. **Simulate the system:** Create a simulation model to track customer arrivals, waiting times, and service times.
4. **Analyze performance:** Calculate metrics like average waiting time, system utilization, and queue length.

2. **Financial Modeling**

Project Goal: Use the exponential distribution to model the time until a financial event (e.g., a market crash, a default) occurs.

Steps:

1. **Determine mean time:** Estimate the average time until the event based on historical data or expert opinion.
2. **Model the time:** Represent the time until the event using an exponential distribution with the estimated mean as the rate parameter.

3. **Calculate probabilities:** Calculate the probability of the event occurring within a given time period.
4. **Risk assessment:** Use the model to assess the risk of the event and its potential impact.

Specific Resources: (web)

OpenIntro Statistics: https://www.openintro.org/book/os/

Unit – 3: Gamma and Beta Distributions (12Hrs)

Gamma distribution – Definition, Moments of Gamma Distribution,Moment generating function, Characteristic function, Cumulant generating function of Gamma distribution. Additive property of Gamma distribution. Measure of skewness, kurtosis and problems. The first four moments are obtained through cumulant generating function,limiting form of Gamma distribution.

Beta Distribution of First Kind and Second kind – Definition, Mean, Variance and Harmonic mean. Simple problems.

Examples/Applications/Case Studies:

Example1: Waiting Time for a Bus for Gamma Distribution

Scenario: A bus arrives at a stop every 15 minutes on average. The time between bus arrivals follows an exponential distribution. What is the probability that you will wait more than 30 minutes for the next bus?

Analysis:

1. The time between bus arrivals is exponentially distributed with a rate parameter $\lambda = 1/15$ (since the average waiting time is 15 minutes).
2. The gamma distribution with shape parameter $\alpha = 1$ and rate parameter $\lambda = 1/15$ is equivalent to the exponential distribution.
3. Therefore, the probability of waiting more than 30 minutes is $P(X > 30) = 1 - F(30)$, where $F(x)$ is the cumulative distribution function of the gamma distribution.

Example 2: Proportion of Defective Items for Beta Distribution

Scenario: A quality control inspector examines a sample of 10 items from a production line. If 3 of the items are defective, what is the probability that the proportion of defective items in the entire population is between 0.2 and 0.4?

Analysis:

1. The proportion of defective items in the population can be modeled using a beta distribution with shape parameters $\alpha = 4$ (number of successes) and $\beta = 7$ (number of failures).

2. The probability of the proportion being between 0.2 and 0.4 is $P(0.2 < X < 0.4) = F(0.4) - F(0.2)$, where $F(x)$ is the cumulative distribution function of the beta distribution.

Exercises/Projects:

Project 1: Modeling Customer Lifetime Value (CLTV) Using the Gamma Distribution

Goal: Use the gamma distribution to model the lifetime value of customers in a business.

Steps:

1. **Collect data:** Gather data on customer purchase history, demographics, and other relevant factors.
2. **Calculate customer lifetime:** Determine the lifetime of each customer based on their purchase history.
3. **Fit a gamma distribution:** Fit a gamma distribution to the customer lifetime data.
4. **Calculate CLTV:** Use the fitted gamma distribution to estimate the expected lifetime value of a customer.
5. **Analyze customer segments:** Identify different customer segments based on their CLTV and analyze their characteristics.

Project 2: Bayesian A/B Testing with Beta Priors

Goal: Use the beta distribution as a prior distribution in Bayesian A/B testing to compare the effectiveness of two different treatments or conditions.

Steps:

1. **Define prior distributions:** Choose appropriate beta priors for the success probabilities of the two treatments based on prior knowledge or assumptions.
2. **Collect data:** Gather data on the number of successes and failures for each treatment.
3. **Update posterior distributions:** Use Bayes' theorem to update the prior distributions based on the observed data.
4. **Calculate Bayes factor:** Calculate the Bayes factor to compare the evidence in favor of one treatment over the other.
5. **Make a decision:** Based on the Bayes factor and other factors, make a decision about whether one treatment is significantly better than the other.

Specific Resources: (web)

https://www.khanacademy.org/math/statistics-probability

Unit – 4: Normal Distribution (12Hrs)

Normal distribution – Definition, Properties of normal distribution, importance of normal distribution, Moment generating function, Characteristic function, Cumulant generating function of normal distribution. Additive property of Normal distribution. Mean, Median and Mode, Even and Odd order moments about mean of normal distribution. Linear combination of normal variates, points of inflexion of normal probability curve. Measure of skewness, and kurtosis.

Standard Normal Distribution – Definition, Moment generating function and Characteristic function, Mean and Variance, Area property and problems.

Examples/Applications/Case Studies:

Case Study 1: Analyzing Student Test Scores

Scenario: A teacher wants to analyze the distribution of test scores for a class of 30 students.

Steps:

1. **Collect data:** Gather the test scores for all 30 students.
2. **Calculate statistics:** Calculate the mean, median, mode, standard deviation, and variance of the test scores.
3. **Visualize the distribution:** Create a histogram and a normal probability plot to assess the normality of the data.
4. **Test for normality:** Use a statistical test, such as the Shapiro-Wilk test or the Kolmogorov-Smirnov test, to formally test for normality.
5. **Analyze results:** If the data is normally distributed, you can use statistical methods based on the normal distribution, such as t-tests or confidence intervals. If the data is not normally distributed, you may need to consider alternative methods, such as nonparametric tests.

Case Study 2: Quality Control in Manufacturing

Scenario: A manufacturing company wants to monitor the quality of a product by measuring its weight. The target weight is 100 grams, and the standard deviation is known to be 5 grams.

Steps:

1. **Collect data:** Measure the weight of a random sample of products.
2. **Calculate statistics:** Calculate the sample mean and standard deviation.
3. **Test for normality:** Use a statistical test to assess the normality of the data.
4. **Construct a confidence interval:** Construct a confidence interval for the population mean weight.

5. **Make a decision:** If the confidence interval includes the target weight of 100 grams, the manufacturing process is considered to be in control. If not, corrective actions may be necessary.

Exercises/Projects:

Project 1: Analyzing Stock Returns

Goal: Analyze the distribution of stock returns and test for normality.

Steps:

1. **Collect data:** Gather historical stock price data for a specific stock or index.
2. **Calculate returns:** Calculate the daily or weekly returns of the stock or index.
3. **Visualize the distribution:** Create a histogram and a normal probability plot to assess the normality of the returns.
4. **Test for normality:** Use a statistical test, such as the Shapiro-Wilk test or the Kolmogorov-Smirnov test, to formally test for normality.
5. **Analyze results:** If the returns are normally distributed, you can use statistical methods based on the normal distribution, such as hypothesis testing or risk analysis. If the returns are not normally distributed, you may need to consider alternative models or techniques.

Project 2: Modeling Customer Satisfaction

Goal: Model customer satisfaction scores using a normal distribution.

Steps:

1. **Collect data:** Gather customer satisfaction scores for a specific product or service.
2. **Analyze the distribution:** Create a histogram and a normal probability plot to assess the normality of the satisfaction scores.
3. **Fit a normal distribution:** Fit a normal distribution to the data to estimate the mean and standard deviation of the satisfaction scores.
4. **Calculate probabilities:** Use the fitted normal distribution to calculate the probability of a customer having a satisfaction score above a certain threshold.
5. **Make inferences:** Use the model to make inferences about customer satisfaction, such as identifying areas for improvement or assessing the impact of changes to the product or service.

Specific Resources: (web)

https://stattrek.com/probability-distributions/poisson.aspx

Unit – 5: Exact Sampling Distributions- (12Hrs)

χ2 - Distribution– Definition, Properties and applications. **Student 's t- distribution–** Definition, Properties and applications. **F – Distribution** – Definition, Properties and applications.

Examples/Applications/Case Studies:

Case Study 1: Sampling Distribution of Sample Mean

Scenario: A researcher wants to estimate the average height of adult males in a city. They randomly sample 100 adult males and measure their heights.

Steps:

1. **Collect data:** Collect the heights of the 100 adult males.
2. **Calculate sample mean:** Calculate the mean height of the sample.
3. **Repeat sampling:** Repeat steps 1 and 2 many times (e.g., 10,000 times) to create a distribution of sample means.
4. **Analyze sampling distribution:** Analyze the distribution of sample means. It should be approximately normal with a mean equal to the population mean and a standard deviation equal to the population standard deviation divided by the square root of the sample size (standard error of the mean).
5. **Make inferences:** Use the sampling distribution to make inferences about the population mean height. For example, construct a confidence interval or conduct a hypothesis test.

Case Study 2: Sampling Distribution of Sample Proportion

Scenario: A political pollster wants to estimate the proportion of voters who support a particular candidate. They randomly sample 500 voters and ask them their preference.

Steps:

1. **Collect data:** Record the number of voters who support the candidate and the total number of voters sampled.
2. **Calculate sample proportion:** Calculate the proportion of voters who support the candidate.
3. **Repeat sampling:** Repeat steps 1 and 2 many times (e.g., 10,000 times) to create a distribution of sample proportions.
4. **Analyze sampling distribution:** Analyze the distribution of sample proportions. It should be approximately normal with a mean equal to the population proportion and a standard deviation equal to the square root of $(p*(1-p))/n$, where p is the population proportion and n is the sample size.

5. **Make inferences:** Use the sampling distribution to make inferences about the population proportion. For example, construct a confidence interval or conduct a hypothesis test.

Exercises/Projects:

Project 1: Simulating the Sampling Distribution of the Sample Mean

Goal: Simulate the sampling distribution of the sample mean from a normally distributed population.

Steps:

1. **Define population parameters:** Specify the population mean (μ) and population standard deviation (σ) of the normal distribution.
2. **Generate random samples:** Draw random samples of a specified size (n) from the normal distribution using the rnorm() function in R.
3. **Calculate sample means:** Calculate the mean of each sample.
4. **Repeat sampling:** Repeat steps 2 and 3 a large number of times (e.g., 10,000) to create a distribution of sample means.
5. **Visualize sampling distribution:** Plot a histogram or density plot of the distribution of sample means.
6. **Compare with theory:** Compare the observed sampling distribution to the theoretical sampling distribution, which is a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.

Project 2: Estimating Population Mean with Confidence Intervals

Goal: Estimate the population mean from a sample and construct confidence intervals using the sampling distribution of the sample mean.

Steps:

1. **Collect data:** Collect a random sample of size n from the population.
2. **Calculate sample mean and standard deviation:** Calculate the sample mean ($\bar{x}$) and sample standard deviation (s) of the data.
3. **Calculate standard error:** Calculate the standard error of the mean using the formula $SE = s/\sqrt{n}$.
4. **Construct confidence intervals:** Construct confidence intervals for the population mean using the t-distribution with n-1 degrees of freedom. For example, a 95% confidence interval can be calculated as $\bar{x} \pm t(\alpha/2, n-1) * SE$, where $t(\alpha/2, n-1)$ is the t-value corresponding to the desired confidence level and degrees of freedom.

5. **Interpret results:** Interpret the confidence intervals and make inferences about the population mean.

Text Books:

1. Gupta. S.C. &Kapoor,V.K. (2023) . Fundamentals of Mathematical Statistics, Sultan Chand & Sons Pvt. Ltd. New Delhi.

References:

1. Bansilal and Arora (1989). New Mathematical Statistics, Satya Prakashan, New Delhi.
2. Goon A.M., Gupta M.K. and Dasgupta B. (2002): Fundamentals of Statistics, Vol. I & II, 8th Edn. The World Press, Kolkata.
3. Mukhopadhyay, P. (2015). Mathematical Statistics.Publisher: BOOKS AND ALLIED (1 January 2016)

SECTION A (5 x 4 = 20 Marks)

(Answer the following questions, internal choice provided. Each question carries 4 marks. Cognitive Level: K2 to K3)

1. **(a)** Define the Continuous Uniform distribution and state its probability density function.

   OR

   (b) Calculate the mean and variance of the Continuous Uniform distribution. (K2)
2. **(a)** Define the Exponential distribution and compute its mean.

   OR

   (b) Define the Double Exponential (Laplace) distribution. State its probability density function and mean.(K2)
3. **(a)** Define the t- distribution and state its applications.

   OR

   (b) Define the F- distribution and state its applications **(K3)**
4. **(a)** State the additive property of the Gamma distribution and provide an example.

   OR

   (b) Define the Beta distribution of the first kind and calculate its mean. (K3)
5. **(a)** Define the Normal distribution and state its importance in statistics.

   OR

   (b) Write down the moment generating function of the Standard Normal distribution.(K3)

SECTION B (5 x 10 = 50 Marks)

(Answer the following questions, internal choice provided. Each question carries 10 marks. Cognitive Level: K3 to K4)

6. **(a)** Derive the moment generating function and characteristic function of the Continuous Uniform distribution and use them to calculate its skewness and kurtosis.(K4)

OR

(b) Compute the median and mean deviation about the mean for the Continuous Uniform distribution.(K4)

7. **(a)** Derive the moment generating function of the Exponential distribution and use it to compute the first four moments. (K4)

OR

(b) Derive the first four moments of the Double Exponential distribution using its characteristic function and compute the skewness and kurtosis. (K4)

8. **(a)** Define the $\chi 2$ distribution, discuss its properties, and explain its applications in statistical testing. (K3)

OR

(b) Obtain the inter-relationship between the t, F and $\chi 2$ distributions. (K4)

9. **(a)** Derive the cumulant generating function of the Gamma distribution and use it to calculate its first four cumulants. (K3)

OR

(b) Explain the Beta distribution of the second kind and calculate its mean and variance. (K3)

10. **(a)** Show that normal distribution is a symmetrical distribution. (K4)

OR

(b) If X,Y are independent normal variates with means 6,7 and variances 9, 16 respectively, λ determine such that $P\left(2X+Y\leq\lambda\right)=P\left(4X-3Y\geq 4\lambda\right)$.(K4)

***